The future in this course.

What’s to come?

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What’s to come? Probability.

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A bag contains:

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A bag contains:

What is the chance that a ball taken from the bag is blue?

The future in this course.

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue.

The future in this course.

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total.

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Count blue. Count total. Divide.

The future in this course.

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Today:

The future in this course.

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide.

Today: Counting!

Outline: basics

1. Counting.
2. Tree
3. Rules of Counting
4. Sample with/without replacement where order does/doesn't matter.

Probability is soon..but first let’s count.

Count?

How many outcomes possible for k coin tosses?
How many poker hands?
How many handshakes for n people?
How many diagonals in a n sided convex polygon?
How many 10 digit numbers?
How many 10 digit numbers without repetition?
How many ways can I divide up 5 dollars among 3 people?

Using a tree..

How many 3-bit strings?

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How many different sequences of three bits from $\{0, 1\}$?

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How would you make one sequence?

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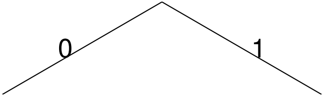

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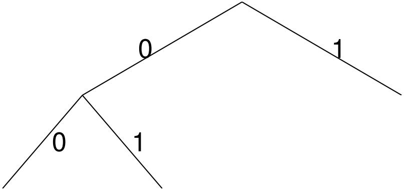

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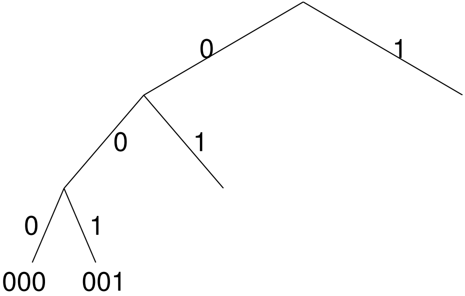

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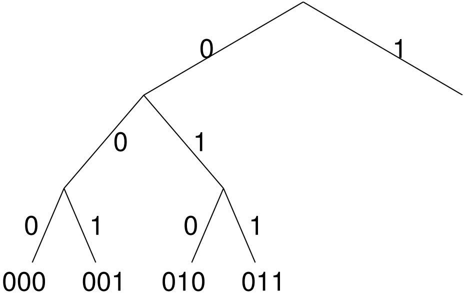

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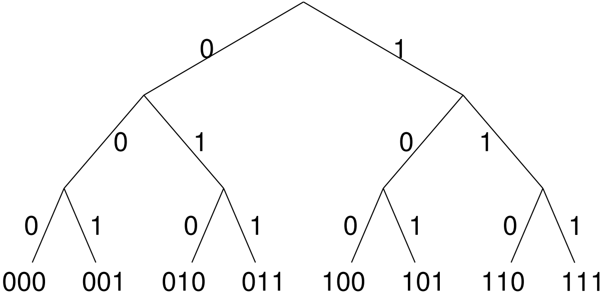

8 leaves which is $2 \times 2 \times 2$.

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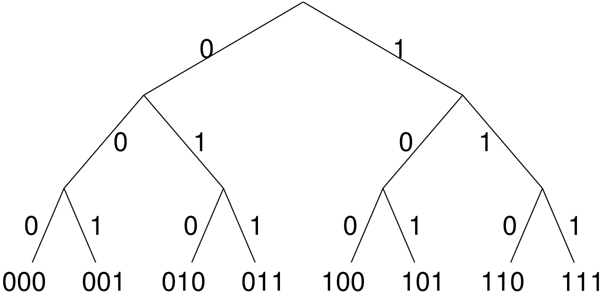

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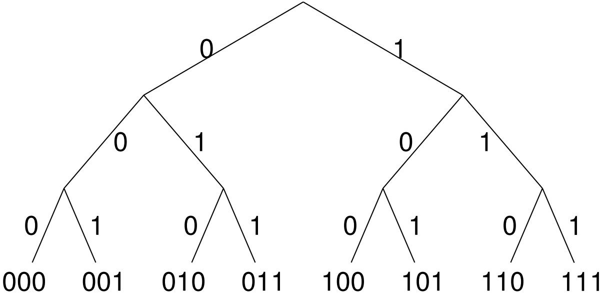

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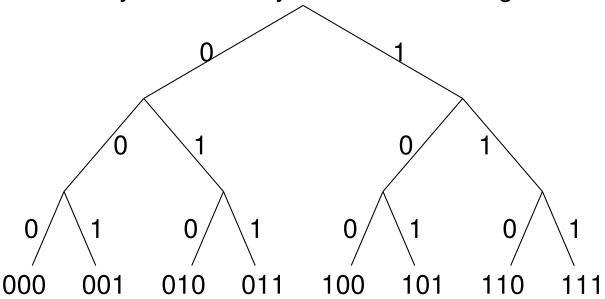

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8 3-bit srings!

First Rule of Counting: Product Rule

Objects made by choosing from n_1, then n_2, $\ldots$, then n_k
the number of objects is $n_1 \times n_2 \cdots \times n_k$.

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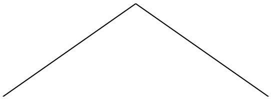

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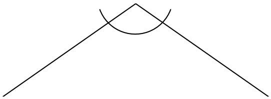

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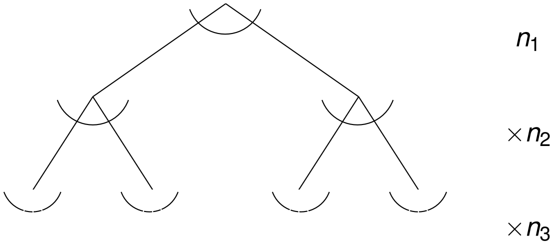

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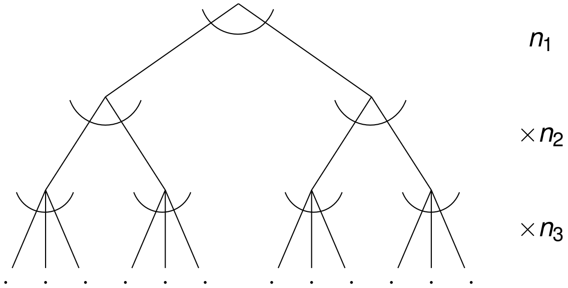

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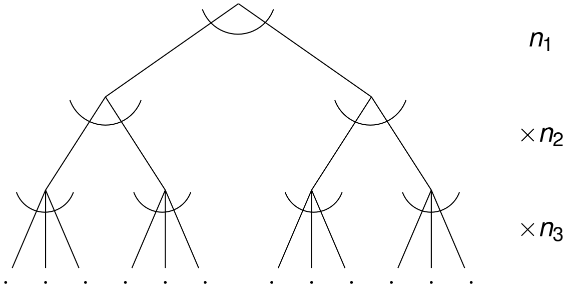

In picture, $2 \times 2 \times 3 = 12$!

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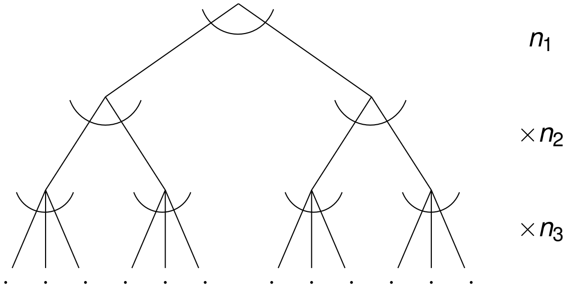

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Using the first rule..

How many outcomes possible for k coin tosses?

Using the first rule..

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2 ways for first choice,

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2 ways for first choice, 2 ways for second choice, ...

Using the first rule..

How many outcomes possible for k coin tosses?

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2×2

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How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots$

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Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
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If no. Then $(m-1)m^{n-1}$.

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How many functions f mapping S to T?

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Questions?

Permutations.

[1]By definition: $0! = 1$.

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How many 10 digit numbers **without repeating a digit**?

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10 ways for first, 9 ways for second,

---

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So total number is $|S| \times |S|-1 \cdots 1 = |S|!$
A one-to-one function is a permutation!

Counting sets..when order doesn't matter.

How many poker hands?

[2] When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$

[2]When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

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$52 \times 51 \times 50 \times 49 \times 48$???

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$52 \times 51 \times 50 \times 49 \times 48$???

Are $A, K, Q, 10, J$ of spades
and $10, J, Q, K, A$ of spades the same?

[2]When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$???

Are $A, K, Q, 10, J$ of spades
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Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.[2]

[2] When each unordered object corresponds equal numbers of ordered objects.

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$52 \times 51 \times 50 \times 49 \times 48$???

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Number of orderings for a poker hand: "5!"

[2]When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$???

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Number of orderings for a poker hand: "5!"
(The "!" means factorial, not Exclamation.)

---

[2] When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$???

Are $A, K, Q, 10, J$ of spades
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Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.[2]

Number of orderings for a poker hand: "5!"

$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$

[2]When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$???

Are $A, K, Q, 10, J$ of spades
and $10, J, Q, K, A$ of spades the same?
Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.[2]

Number of orderings for a poker hand: "5!"

Can write as...

$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$

$$\frac{52!}{5! \times 47!}$$

[2] When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$???

Are $A, K, Q, 10, J$ of spades
and $10, J, Q, K, A$ of spades the same?
Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.[2]

Number of orderings for a poker hand: "5!"

Can write as...
$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$
$$\frac{52!}{5! \times 47!}$$

Generic: ways to choose 5 out of 52 possibilities.

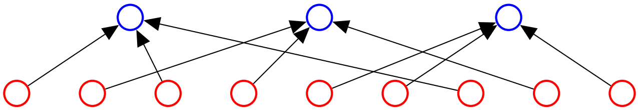
[2] When each unordered object corresponds equal numbers of ordered objects.

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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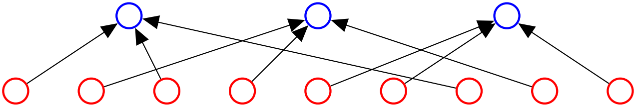
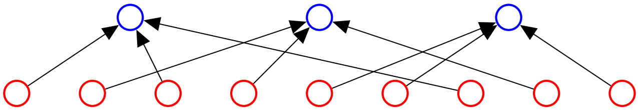


How many red nodes (ordered objects)?

Ordered to unordered.

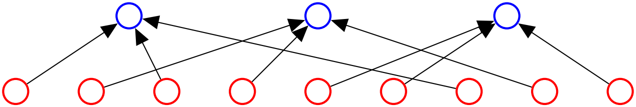
Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



How many red nodes (ordered objects)? 9.

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.

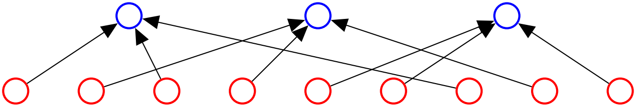


How many red nodes (ordered objects)? 9.

How many red nodes mapped to one blue node?

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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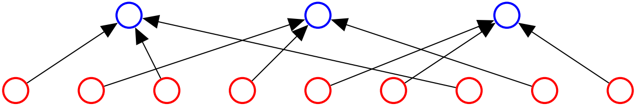

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How many blue nodes (unordered objects)?

Ordered to unordered.

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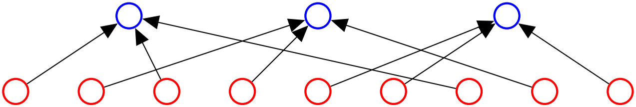


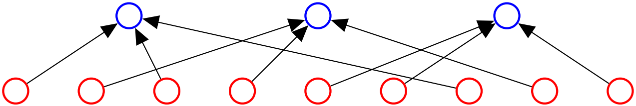
How many red nodes (ordered objects)? 9.

How many red nodes mapped to one blue node? 3.

How many blue nodes (unordered objects)? $\frac{9}{3}$

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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Ordered to unordered.

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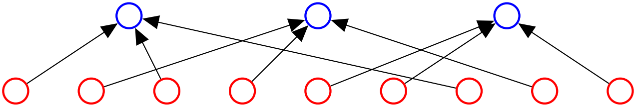

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How many poker deals?

Ordered to unordered.

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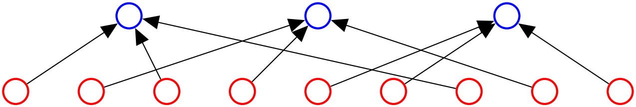

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How many poker deals? $52 \cdot 51 \cdot 50 \cdot 49 \cdot 48$.

Ordered to unordered.

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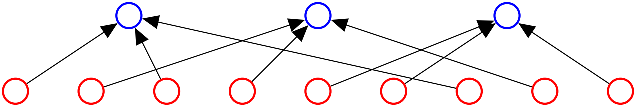

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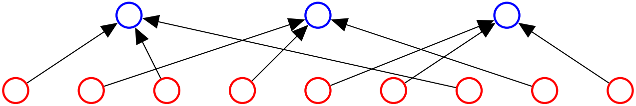
How many poker deals per hand?

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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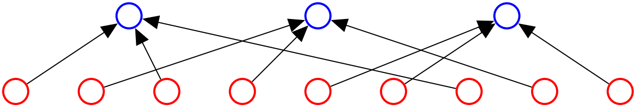
How many poker deals per hand?
Map each deal to ordered deal:

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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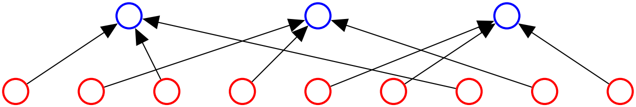
How many poker deals? $52 \cdot 51 \cdot 50 \cdot 49 \cdot 48$.

How many poker deals per hand?
Map each deal to ordered deal: $5!$

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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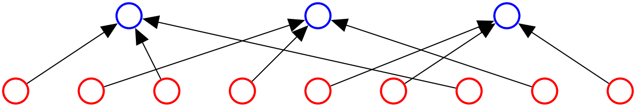

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How many poker deals per hand?
Map each deal to ordered deal: $5!$

How many poker hands? $\frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5!}$

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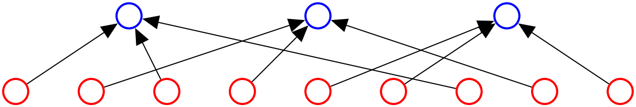

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Questions?

..order doesn’t matter.

..order doesn't matter.

Choose 2 out of n?

..order doesn't matter.

Choose 2 out of n?

$$\frac{n \times (n-1)}{}$$

..order doesn't matter.

Choose 2 out of n?

$$\frac{n \times (n-1)}{2}$$

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Choose 2 out of n?

$$\frac{n \times (n-1)}{2} = \frac{n!}{(n-2)! \times 2}$$

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Choose 3 out of n?

..order doesn't matter.

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$$\frac{n \times (n-1) \times (n-2)}{3!} = \frac{n!}{(n-3)! \times 3!}$$

Choose k **out of** n?

$$\frac{n!}{(n-k)!\qquad\qquad}$$

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Choose k **out of** n?

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Notation: $\binom{n}{k}$ **and pronounced "**n **choose** k**."**

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Familiar?

..order doesn't matter.

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Choose 3 out of n?

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Choose k **out of** n?

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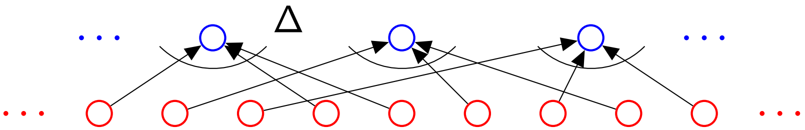
Notation: $\binom{n}{k}$ **and pronounced "**n **choose** k**."**

Familiar? Questions?

Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$. **Product Rule.**
Second rule: when order doesn't matter divide...



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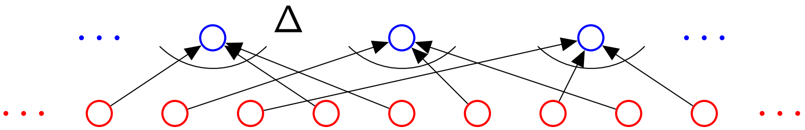
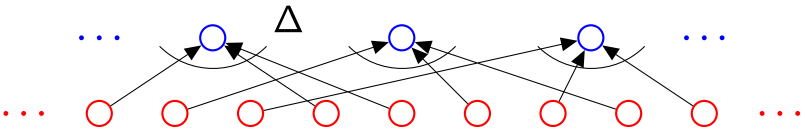


3 card Poker deals: 52

Example: Visualize the proof..

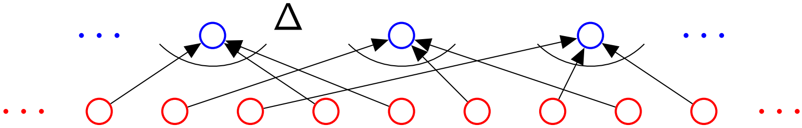
First rule: $n_1 \times n_2 \cdots \times n_3$**. Product Rule.**
Second rule: when order doesn't matter divide...



3 card Poker deals: 52×51

Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$**. Product Rule.**
Second rule: when order doesn't matter divide...



3 card Poker deals: $52 \times 51 \times 50$

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First rule: $n_1 \times n_2 \cdots \times n_3$. **Product Rule.**
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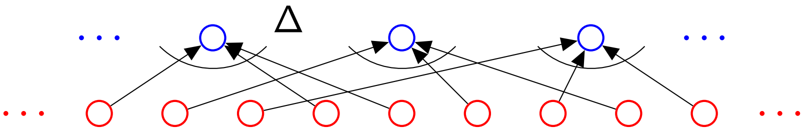

3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$.

Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$**. Product Rule.**
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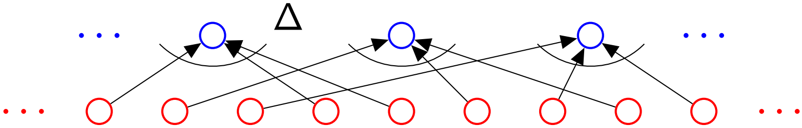

3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.

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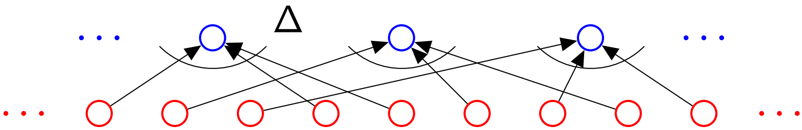

3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.
Poker hands: Δ?

Example: Visualize the proof..

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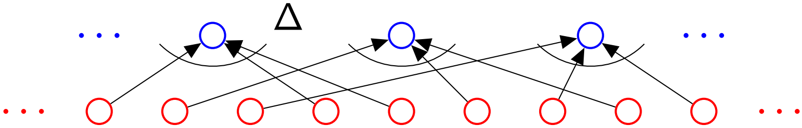
Second rule: when order doesn't matter divide...



3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.
Poker hands: Δ?
Hand: Q, K, A.

Example: Visualize the proof..

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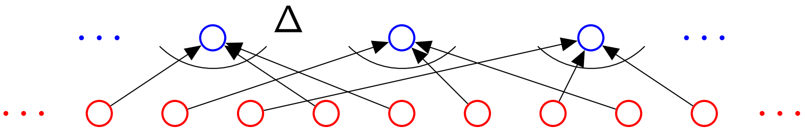

3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.
Poker hands: Δ?
Hand: Q, K, A.
Deals: Q, K, A :

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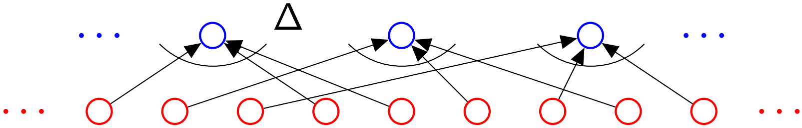

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Hand: Q, K, A.
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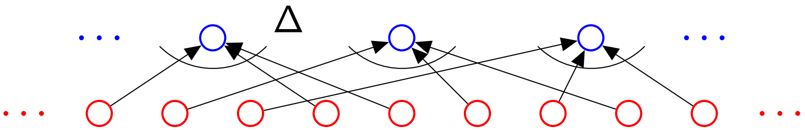

3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.
Poker hands: Δ?
Hand: Q, K, A.
Deals: $Q, K, A : Q, A, K : K, A, Q : K, A, Q : A, K, Q : A, Q, K$.

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First rule: $n_1 \times n_2 \cdots \times n_3$**. Product Rule.**
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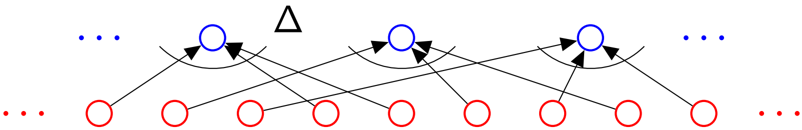


3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.
Poker hands: Δ?

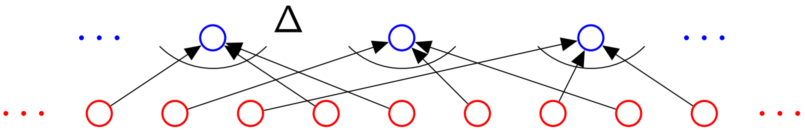
Hand: Q, K, A.

Deals: $Q, K, A : Q, A, K : K, A, Q : K, A, Q : A, K, Q : A, Q, K$.

$\Delta = 3 \times 2 \times 1$

Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$. **Product Rule.**
Second rule: when order doesn't matter divide...



3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.
Poker hands: Δ?
Hand: Q, K, A.
Deals: $Q, K, A : Q, A, K : K, A, Q : K, A, Q : A, K, Q : A, Q, K$.
$\Delta = 3 \times 2 \times 1$ First rule again.

Example: Visualize the proof..

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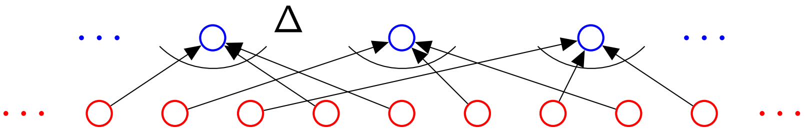

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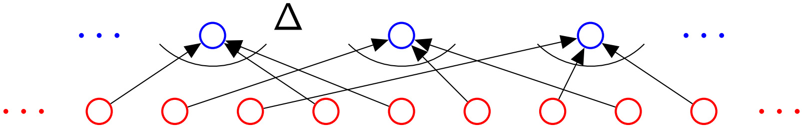

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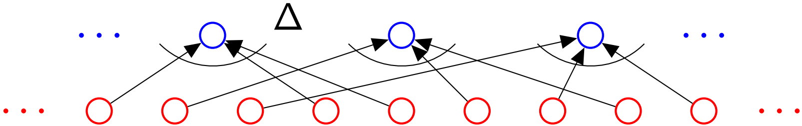

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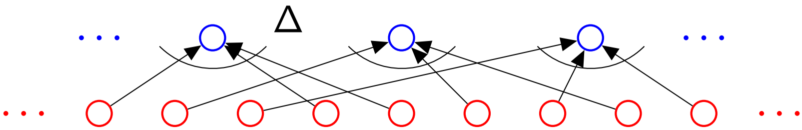

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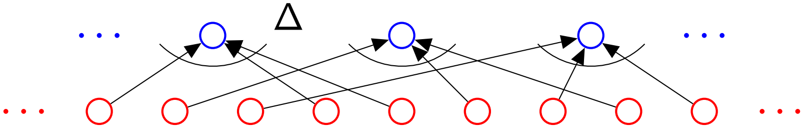

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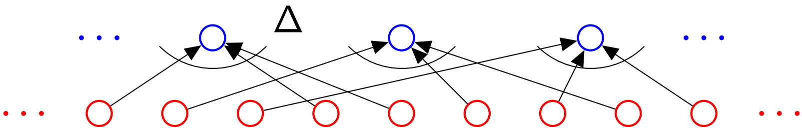

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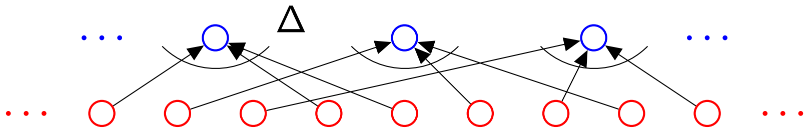

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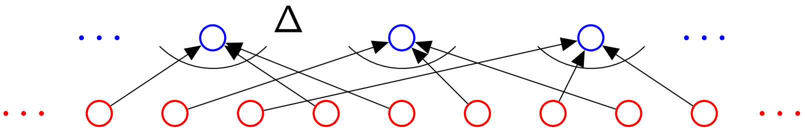

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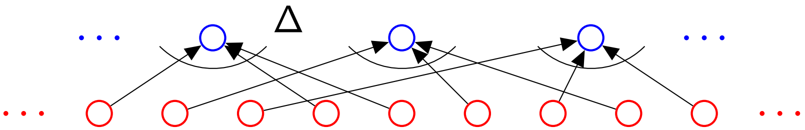

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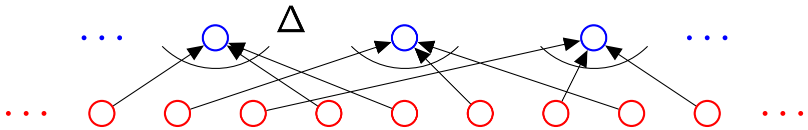

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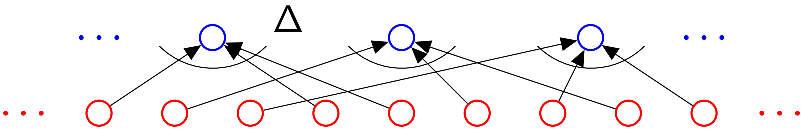

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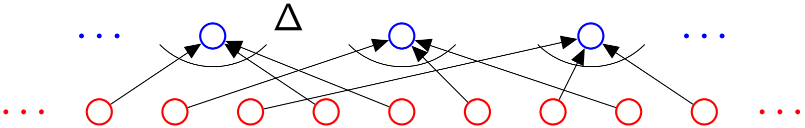

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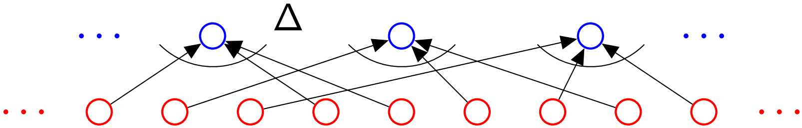

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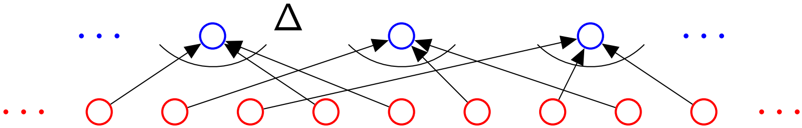

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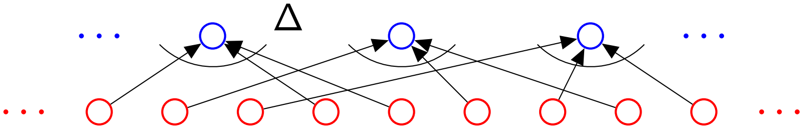

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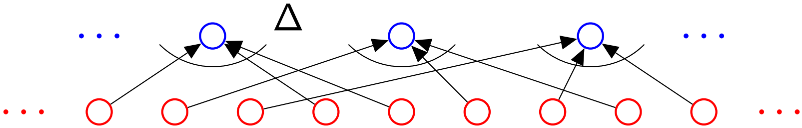

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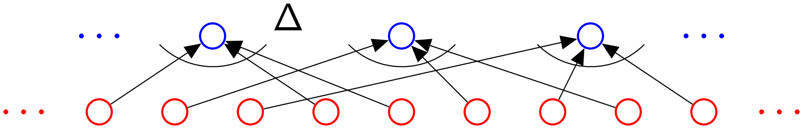

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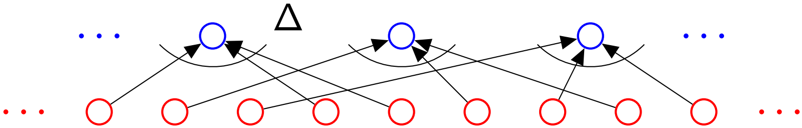

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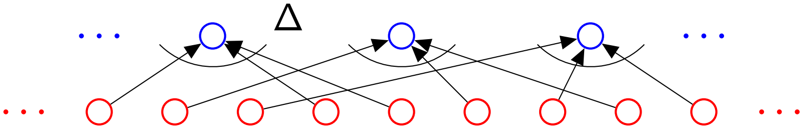

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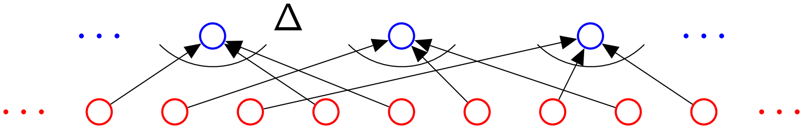

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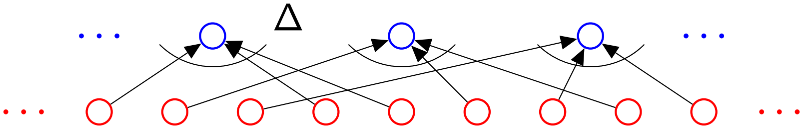

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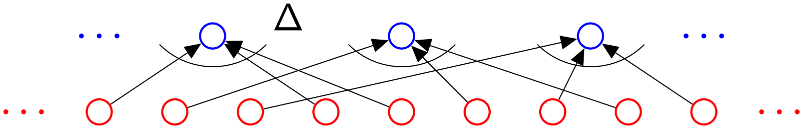

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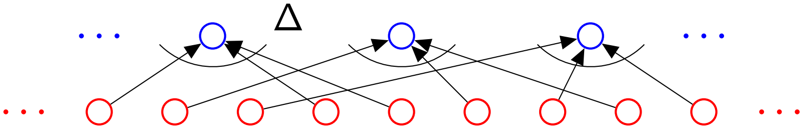

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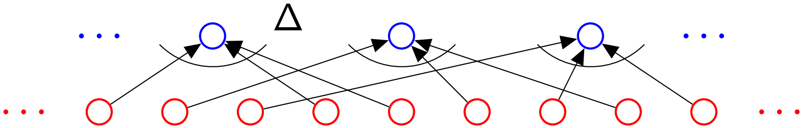

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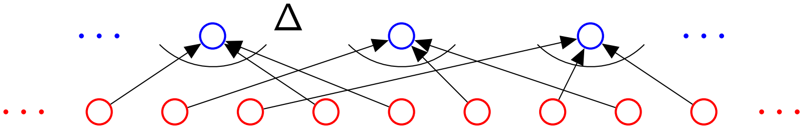

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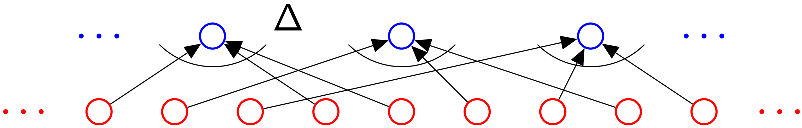

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Some Practice.

How many orderings of letters of CAT?

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3 ways to choose first letter, 2 ways for second, 1 for last.

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How many orderings of MISSISSIPPI?

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How many orderings of MISSISSIPPI?

4 S's, 4 I's, 2 P's.

Some Practice.

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How many orderings of MISSISSIPPI?

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11 letters total.

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Total orderings? $\frac{7!}{3!}$

How many orderings of MISSISSIPPI?

4 S’s, 4 I’s, 2 P’s.
11 letters total.
11! ordered objects.

Some Practice.

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How many orderings of MISSISSIPPI?

4 S’s, 4 I’s, 2 P’s.
11 letters total.
11! ordered objects.
$4! \times 4! \times 2!$ ordered objects per “unordered object”

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total orderings of 7 letters. 7!
total "extra counts" or orderings of three A's? 3!

Total orderings? $\frac{7!}{3!}$

How many orderings of MISSISSIPPI?

4 S's, 4 I's, 2 P's.
11 letters total.
$11!$ ordered objects.
$4! \times 4! \times 2!$ ordered objects per "unordered object"

$\implies \frac{11!}{4!4!2!}$.

Sampling...

Sample k items out of n

Sampling...

Sample k items out of n

Without replacement:

Sampling...

Sample k items out of n

Without replacement:

Order matters:

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

Second Rule: divide by number of orders

Sampling...

Sample k items out of n

- Without replacement:
  - Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$
  - Order does not matter:
    - Second Rule: divide by number of orders – "$k!$"

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

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Second Rule: divide by number of orders – "k!"

$\implies \frac{n!}{(n-k)!k!}$.

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- Without replacement:
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      - $\implies \frac{n!}{(n-k)!k!}$.
  - "n choose k"

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Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

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With Replacement.

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Order matters: n

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

Second Rule: divide by number of orders – "$k!$"

$\implies \frac{n!}{(n-k)!k!}$.

"n choose k"

With Replacement.

Order matters: $n \times n$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

Second Rule: divide by number of orders – "$k!$"

$\Longrightarrow \frac{n!}{(n-k)!k!}$.

"n choose k"

With Replacement.

Order matters: $n \times n \times \ldots n$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

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With Replacement.

Order matters: $n \times n \times \ldots n = n^k$

Sampling...

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Order does not matter: Second rule

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Problem: depends on how many of each item we chose!

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Different number of unordered elts map to each unordered elt.

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Order does not matter: Second rule ???

Problem: depends on how many of each item we chose!

Different number of unordered elts map to each unordered elt.

Unordered elt: $1, 2, 3$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

Second Rule: divide by number of orders – “$k!$”

$\implies \frac{n!}{(n-k)!k!}$.

“n choose k”

With Replacement.

Order matters: $n \times n \times \ldots n = n^k$

Order does not matter: Second rule ???

Problem: depends on how many of each item we chose!

Different number of unordered elts map to each unordered elt.

Unordered elt: $1,2,3$ 3! ordered elts map to it.

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

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Unordered elt: $1,2,2$ $\frac{3!}{2!}$ ordered elts map to it.

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Second Rule: divide by number of orders – "k!"

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Order matters: $n \times n \times \ldots n = n^k$

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Problem: depends on how many of each item we chose!

Different number of unordered elts map to each unordered elt.

Unordered elt: $1, 2, 3$ 3! ordered elts map to it.
Unordered elt: $1, 2, 2$ $\frac{3!}{2!}$ ordered elts map to it.

How do we deal with this

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Sample k items out of n

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Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$
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Second Rule: divide by number of orders – "$k!$"
$\implies \frac{n!}{(n-k)!k!}$.
"n choose k"

With Replacement.
Order matters: $n \times n \times \ldots n = n^k$
Order does not matter: Second rule ???

Problem: depends on how many of each item we chose!
Different number of unordered elts map to each unordered elt.

Unordered elt: $1,2,3$ $3!$ ordered elts map to it.
Unordered elt: $1,2,2$ $\frac{3!}{2!}$ ordered elts map to it.

How do we deal with this mess??

Splitting up some money....

How many ways can Bob and Alice split 5 dollars?

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For each of 5 dollars pick Bob or Alice(2^5), divide out order

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5 dollars for Bob and 0 for Alice:

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How many ways can Bob and Alice split 5 dollars?
For each of 5 dollars pick Bob or Alice(2^5), divide out order ???

5 dollars for Bob and 0 for Alice:
one ordered set: (B, B, B, B, B).

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How many ways can Bob and Alice split 5 dollars?
For each of 5 dollars pick Bob or Alice(2^5), divide out order ???

5 dollars for Bob and 0 for Alice:
one ordered set: (B, B, B, B, B).

4 for Bob and 1 for Alice:

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5 dollars for Bob and 0 for Alice:
one ordered set: (B, B, B, B, B).

4 for Bob and 1 for Alice:
5 ordered sets: (A, B, B, B, B) ; (B, A, B, B, B); ...

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How many ways can Bob and Alice split 5 dollars?
For each of 5 dollars pick Bob or Alice(2^5), divide out order ???

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"Sorted" way to specify, first Alice's dollars, then Bob's.

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“Sorted” way to specify, first Alice’s dollars, then Bob’s.

(B,B,B,B,B):
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(A,A,B,B,B):

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How many ways can Bob and Alice split 5 dollars?
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5 dollars for Bob and 0 for Alice:
one ordered set: (B, B, B, B, B).

4 for Bob and 1 for Alice:
5 ordered sets: (A, B, B, B, B) ; (B, A, B, B, B); ...

"Sorted" way to specify, first Alice's dollars, then Bob's.

(B, B, B, B, B):
(A, B, B, B, B):
(A, A, B, B, B):

and so on.

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one ordered set: (B, B, B, B, B).

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5 ordered sets: (A, B, B, B, B) ; (B, A, B, B, B); ...

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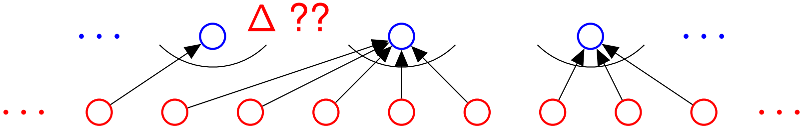
(B, B, B, B, B):
(A, B, B, B, B):
(A, A, B, B, B):

and so on.



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5 dollars for Bob and 0 for Alice:
one ordered set: (B, B, B, B, B).

4 for Bob and 1 for Alice:
5 ordered sets: (A, B, B, B, B) ; (B, A, B, B, B); ...

"Sorted" way to specify, first Alice's dollars, then Bob's.

(B, B, B, B, B): 1: (B,B,B,B,B)
(A, B, B, B, B):
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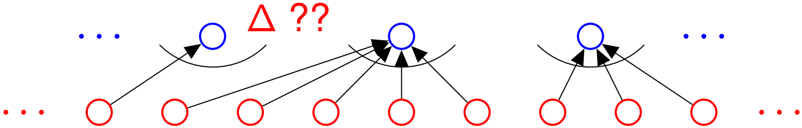


Splitting up some money....

How many ways can Bob and Alice split 5 dollars?
For each of 5 dollars pick Bob or Alice(2^5), divide out order ???

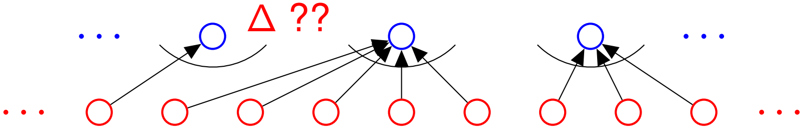
5 dollars for Bob and 0 for Alice:
one ordered set: (B, B, B, B, B).

4 for Bob and 1 for Alice:
5 ordered sets: (A, B, B, B, B) ; (B, A, B, B, B); ...

"Sorted" way to specify, first Alice's dollars, then Bob's.

(B, B, B, B, B): 1: (B,B,B,B,B)
(A, B, B, B, B): 5: (A,B,B,B,B),(B,A,B,B,B),(B,B,A,B,B),...
(A, A, B, B, B):

and so on.



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For each of 5 dollars pick Bob or Alice(2^5), divide out order ???

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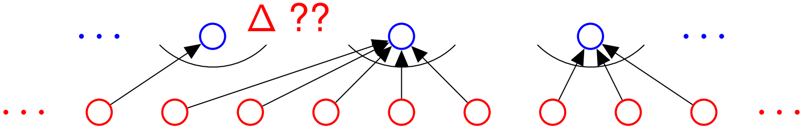
one ordered set: (B, B, B, B, B).

4 for Bob and 1 for Alice:
5 ordered sets: (A, B, B, B, B) ; (B, A, B, B, B); ...

"Sorted" way to specify, first Alice's dollars, then Bob's.

(B, B, B, B, B): 1: (B,B,B,B,B)
(A, B, B, B, B): 5: (A,B,B,B,B),(B,A,B,B,B),(B,B,A,B,B),...
(A, A, B, B, B): $\binom{5}{2}$;(A,A,B,B,B),(A,B,A,B,B),(A,B,B,A,B),...

and so on.



Second rule of counting is no good here!

Splitting 5 dollars..

How many ways can Alice, Bob, and Eve split 5 dollars.

Splitting 5 dollars..

How many ways can Alice, Bob, and Eve split 5 dollars.

Alice gets 3, Bob gets 1, Eve gets 1: (A, A, A, B, E).

Splitting 5 dollars..

How many ways can Alice, Bob, and Eve split 5 dollars.

Alice gets 3, Bob gets 1, Eve gets 1: (A, A, A, B, E).

Separate Alice's dollars from Bob's and then Bob's from Eve's.

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How many ways can Alice, Bob, and Eve split 5 dollars.

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Five dollars are five stars: $\star\star\star\star\star$.

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Alice: 2, Bob: 1, Eve: 2.

Splitting 5 dollars..

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Alice gets 3, Bob gets 1, Eve gets 1: (A, A, A, B, E).

Separate Alice's dollars from Bob's and then Bob's from Eve's.

Five dollars are five stars: $\star\star\star\star\star$.

Alice: 2, Bob: 1, Eve: 2.
Stars and Bars: $\star\star|\star|\star\star$.

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How many ways can Alice, Bob, and Eve split 5 dollars.

Alice gets 3, Bob gets 1, Eve gets 1: (A, A, A, B, E).

Separate Alice's dollars from Bob's and then Bob's from Eve's.

Five dollars are five stars: $\star\star\star\star\star$.

Alice: 2, Bob: 1, Eve: 2.
Stars and Bars: $\star\star|\star|\star\star$.

Alice: 0, Bob: 1, Eve: 4.

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Stars and Bars: $\star\star|\star|\star\star$.

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Each split "is" a sequence of stars and bars.

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Stars and Bars: $|\star|\star\star\star\star$.

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Each sequence of stars and bars "is" a split.

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How many ways can Alice, Bob, and Eve split 5 dollars.

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Stars and Bars: $|\star|\star\star\star\star$.

Each split "is" a sequence of stars and bars.
Each sequence of stars and bars "is" a split.

Counting Rule: if there is a one-to-one mapping between two sets they have the same size!

Stars and Bars.

How many different 5 star and 2 bar diagrams?

Stars and Bars.

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$| \star | \star \star \star \star$.

Stars and Bars.

How many different 5 star and 2 bar diagrams?

$| \star | \star \star \star \star$.

7 positions in which to place the 2 bars.

$\_\ \_\ \_\ \_\ \_\ \_\ \_$

Stars and Bars.

How many different 5 star and 2 bar diagrams?

$| \star | \star \star \star \star$.

7 positions in which to place the 2 bars.

$\_\ \_\ \_\ \_\ \_\ \_\ \_$

Alice: 0; Bob 1; Eve: 4

Stars and Bars.

How many different 5 star and 2 bar diagrams?

$| \star | \star \star \star \star$.

7 positions in which to place the 2 bars.

$\_ \_ \_ \_ \_ \_ \_$

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$| \star | \star \star \star \star$.

Stars and Bars.

How many different 5 star and 2 bar diagrams?

$| \star | \star \star \star \star$.

7 positions in which to place the 2 bars.

$\_ \; \_ \; \_ \; \_ \; \_ \; \_ \; \_$

Alice: 0; Bob 1; Eve: 4
$| \star | \star \star \star \star$.
Bars in first and third position.

Stars and Bars.

How many different 5 star and 2 bar diagrams?

$| \star | \star \star \star \star$.

7 positions in which to place the 2 bars.

$\_ \_ \_ \_ \_ \_ \_$

Alice: 0; Bob 1; Eve: 4
$| \star | \star \star \star \star$.
Bars in first and third position.

Alice: 1; Bob 4; Eve: 0

Stars and Bars.

How many different 5 star and 2 bar diagrams?

$| \star | \star \star \star \star$.

7 positions in which to place the 2 bars.

$\_ \; \_ \; \_ \; \_ \; \_ \; \_ \; \_$

Alice: 0; Bob 1; Eve: 4
$| \star | \star \star \star \star$.
Bars in first and third position.

Alice: 1; Bob 4; Eve: 0
$\star | \star \star \star \star |$.

Stars and Bars.

How many different 5 star and 2 bar diagrams?

$| \star | \star \star \star \star$.

7 positions in which to place the 2 bars.

$\_ \_ \_ \_ \_ \_ \_$

Alice: 0; Bob 1; Eve: 4
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Bars in first and third position.

Alice: 1; Bob 4; Eve: 0
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Bars in second and seventh position.

Stars and Bars.

How many different 5 star and 2 bar diagrams?

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Or: k unordered choices from set of n possibilities with replacement.
Sample with replacement where order doesn't matter.

Summary.

First rule: $n_1 \times n_2 \cdots \times n_3$.

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Sample k times from n objects with replacement and order doesn't matter: $\binom{k+n-1}{n-1}$.

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