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Recall: powerset of the naturals is not countable.

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Uh oh....

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Axioms changed.

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See Logicomix by Doxiaidis, Papadimitriou (was professor here), Papadatos, Di Donna.

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Write me a program checker!

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Something about infinity here, maybe?

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Halt and Turing.

Proof:

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Proof: Assume there is a program $HALT(\cdot, \cdot)$.

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2. Otherwise, halt immediately.

Assumption: there is a program HALT.
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Can run Turing on Turing!

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Another view of proof: diagonalization.

Any program is a fixed length string.

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P_2	L	L	H	$\cdots$
P_3	L	H	H	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

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Assumed HALT(P, I) existed.

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We are so smart!

Wow, that was easy!

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Wow, that was easy!

We should be famous!

No computers for Turing!

In Turing's time.

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No computers.

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No computers.

Adding machines.

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e.g., Babbage (from table of logarithms) 1812.

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Adding machines.
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Concept of program as data wasn't really there.

Turing machine.

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A Turing machine.
– an (infinite) tape with characters

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– be in a state, and read a character

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Turing: AI, self modifying code, learning...

Turing and computing.

Just a mathematician?

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Simulated the program by hand to play chess.

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Church, Gödel and Turing.

Church proved an equivalent theorem. (Previously.)

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Programming languages!

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Programming languages! javascript, ruby, python....

Gödel: Incompleteness theorem.

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Computing on top of computing...

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We can't get enough of building more Turing machines.

Undecidable problems.

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Undecidability for Diophantine set of equations
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Tragic ending...

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- British Government apologized (2009) and pardoned (2013).

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Program is text, so we can pass it to itself,
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Summary: decidability.

Computer Programs are an interesting thing.

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Computation is a lens for other action in the world.

Probability

What’s to come?

Probability

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Probability

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A bag contains:

Probability

What's to come? Probability.

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Probability

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A bag contains:

What is the chance that a ball taken from the bag is blue?

Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue.

Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total.

Probability

What's to come? Probability.

A bag contains:

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Count blue. Count total. Divide.

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What's to come? Probability.

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Next Up: Probability.