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Integration by Parts: $\int udv = uv - \int vdu$.

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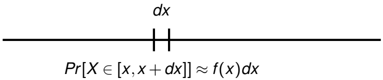

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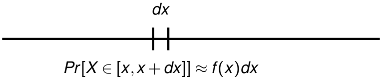

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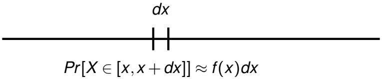

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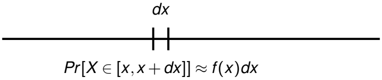

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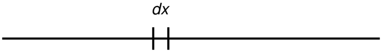

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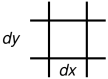

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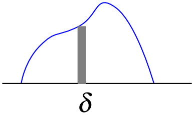

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Total Probability Rule: $Pr[A] = Pr[A \cap B] + Pr[A \cap \overline{B}]$
Pr["Second Heads"] $= Pr[HH] + Pr[HT]$
B is First coin heads.
$Pr[X \in [.45, .55]] = Pr[X \in [.45, .50]] + Pr[X \in (.50, .55]]$
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Product Rule: $Pr[A \cap B] = Pr[A|B]Pr[B]$.
Bayes Rule: $Pr[A|B] = Pr[B|A]Pr[A]/Pr[B]$.

Probability Rules are all good.

Conditional Probabity.

Events: A, B

Discrete: "Heads", "Tails", $X = 1$, $Y = 5$.

Continuous: X in $[.2, .3]$. $X \in [.2, .3]$ or $X \in [.4, .6]$.

Conditional Probability: $Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]}$

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All work for continuous with intervals as events.

Expo(λ)

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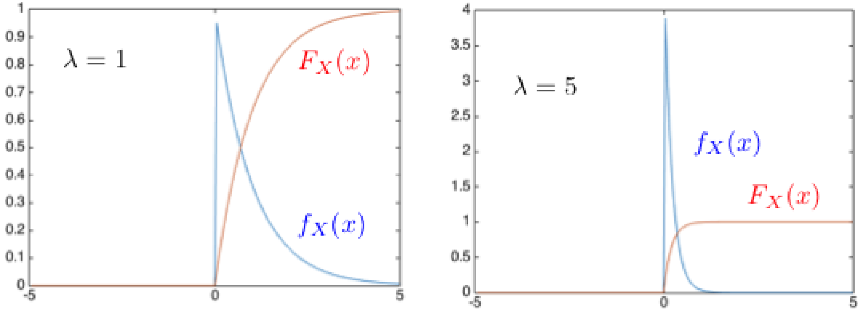

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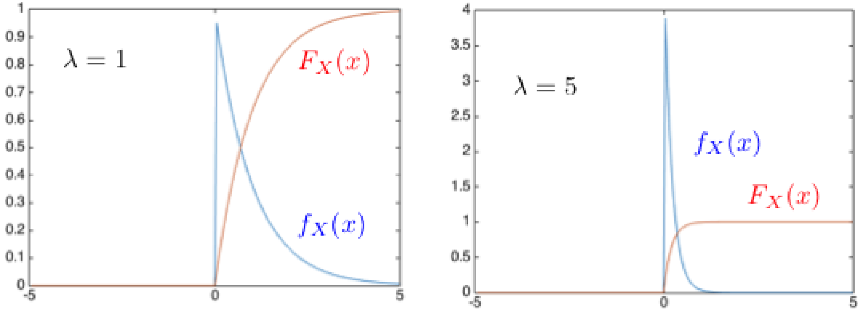

Note that $Pr[X > t] = e^{-\lambda t}$ for $t > 0$.

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Replace b by $b - a$, use $X = U[0,1]$, then $Y = a + (b-a)X$ is $U[a,b]$.

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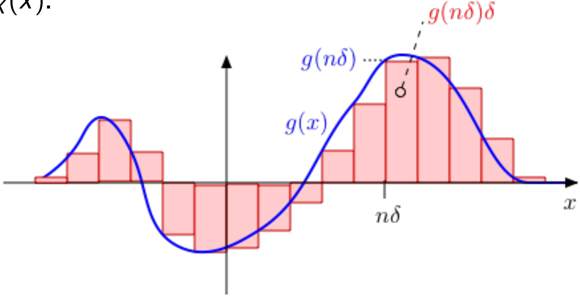

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Hence, $E[X] = \frac{1}{\lambda}$.

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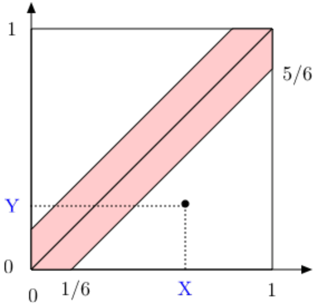

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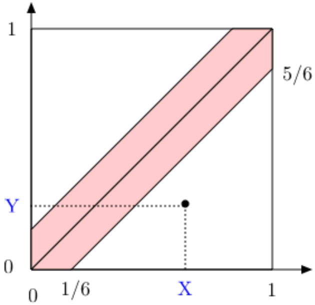

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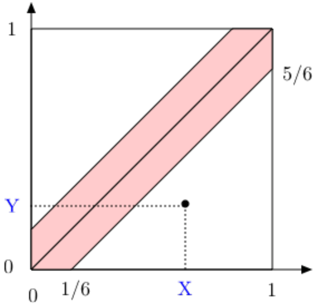

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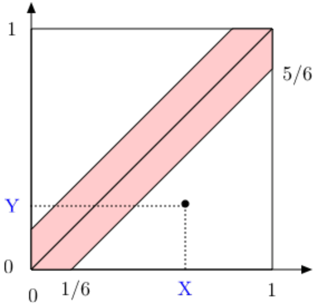

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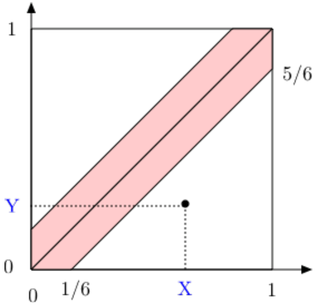

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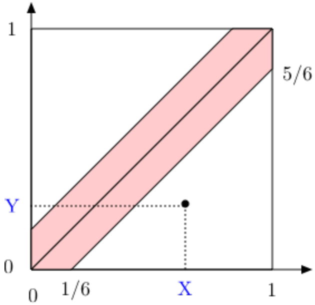

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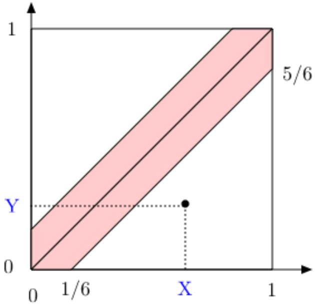

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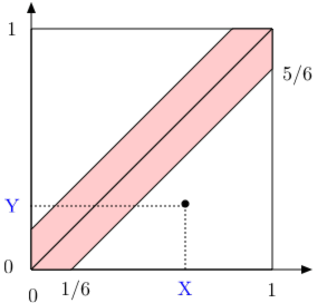

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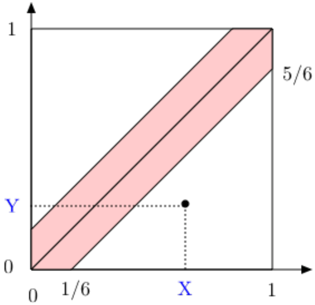

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Thus, $Pr[\text{meet}] = 1 - (\frac{5}{6})^2 = \frac{11}{36}$.

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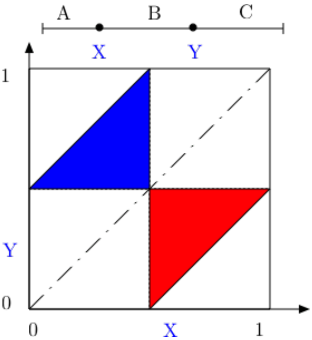

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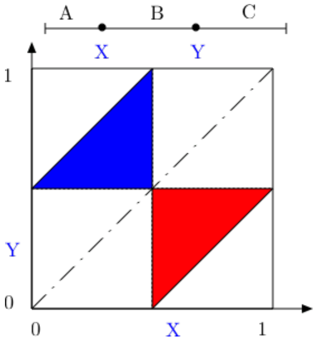

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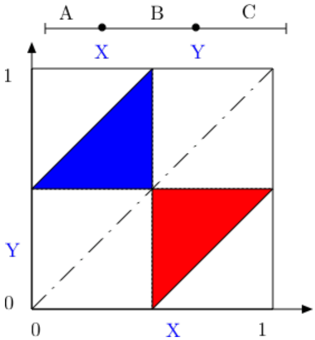

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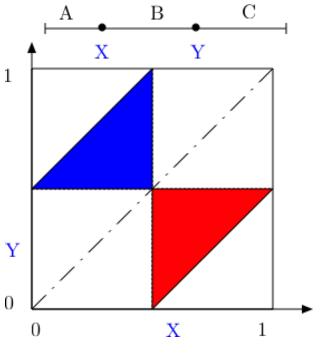

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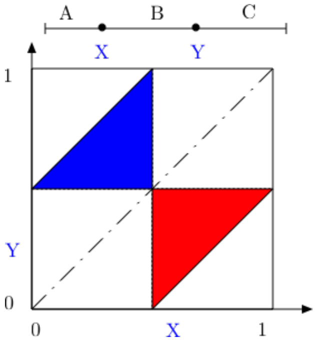

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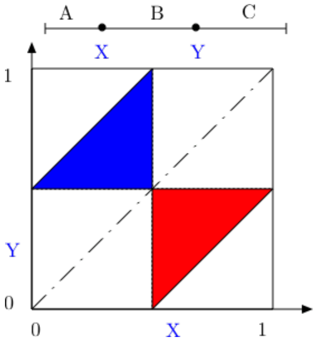

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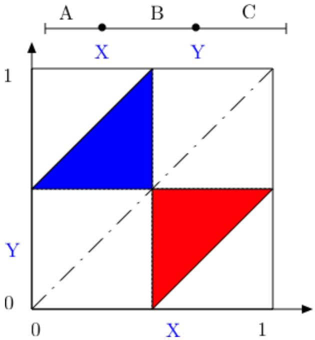

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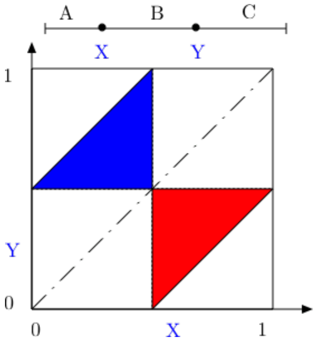

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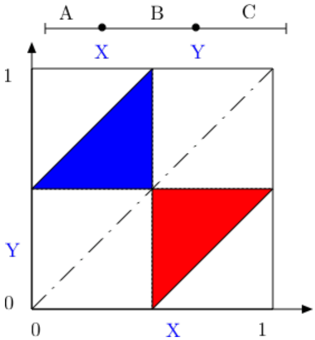

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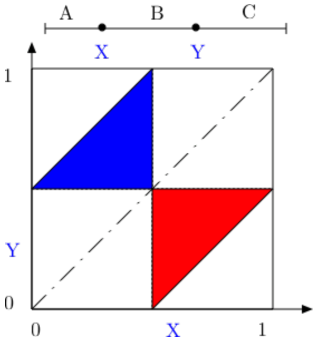

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Thus, $Pr[\text{make triangle}] = 1/4$.

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Maximum of Two Exponentials

Let $X = Expo(\lambda)$ and $Y = Expo(\mu)$ be independent.
Define $Z = \max\{X, Y\}$.

Calculate $E[Z]$.

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Hence,

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For instance, if $n = 16$, then $SNR(dB) \approx 112dB$.

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Indeed, $(1 - \frac{a}{N})^N \approx \exp\{-a\}$.

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