Review.

After Midterm content.

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See lecture 13, for pre-Midterm review.

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CS70: Review of Probability.

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- WLLN: X_m i.i.d. $\Rightarrow \frac{X_1 + \cdots + X_n}{n} \approx E[X]$

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6. Joint pdf: $Pr[X \in (x, x+\delta), Y = (y, y+\delta)) = f_{X,Y}(x,y)\delta^2$.
   6.1 Conditional Distribution: $f_{X|Y}(x,y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$.
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- $X \sim N(\mu, \sigma)$. $f(x) = \frac{1}{\sqrt{2\pi}} e^{-(x-\mu)^2} 2 \quad F(x) = \Phi(x) = \int_{-\infty}^{x} f(x) dx$
  CLT: $A_n = \frac{X_1 + \cdots X_n}{n}$, $E[X] = \mu$, and $Var(X) = \sigma^2$.
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- Irreducible + Aperiodic $\Rightarrow \pi_n \to \pi$.
- Calculating π: One finds $P\pi = \pi$ and π is distribution.

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6. Common Mistakes

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Match Items

[1] $Pr[X \geq z] \leq \frac{E[f(X)]}{f(a)}$.

[2] $Pr[|X - E[X]| \geq z] \leq \frac{Var(X)}{z^2}$.

[3] $\sum_y yPr[Y = y|X = x]$.

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- Markov's inequality (1)
- $E[Y|X = x]$ (3)

Quiz 1: G

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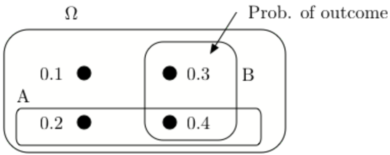

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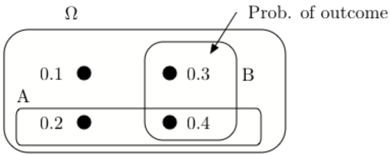

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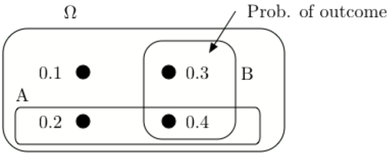

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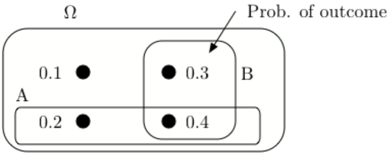

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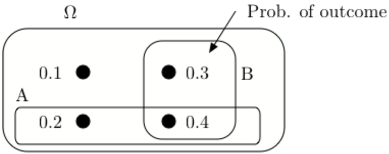

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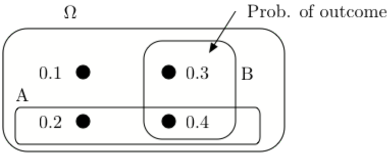

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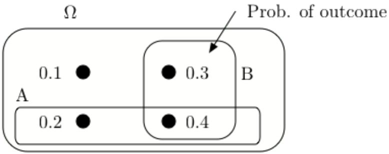

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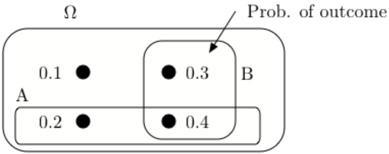

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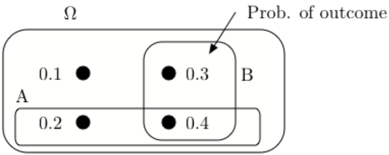

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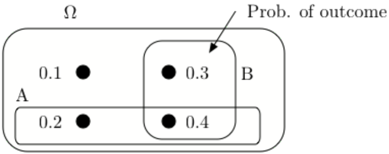

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3. Are A and B positively correlated?

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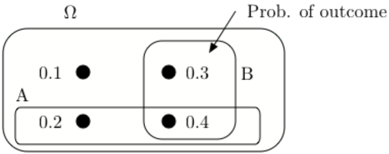

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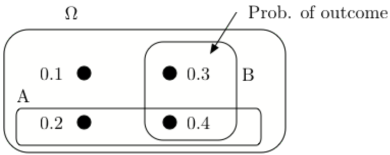

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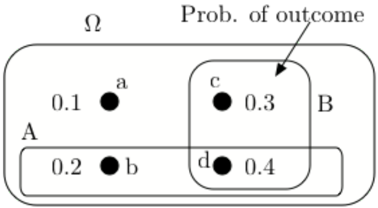

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b	1	0
c	0	2
d	1	2

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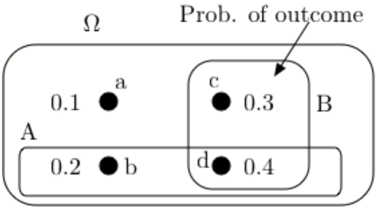

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4. What is $cov(X, Y)$?

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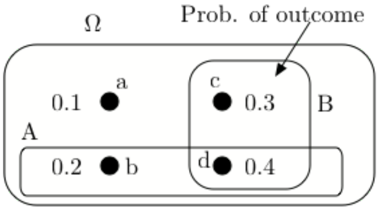

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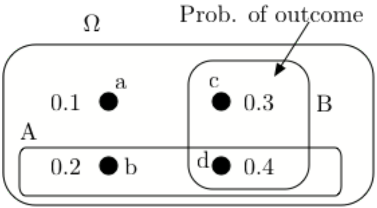
4. What is $cov(X, Y)$?
   $cov(X, Y) =$

Quiz 1: G



ω	$X(\omega)$	$Y(\omega)$
a	0	0
b	1	0
c	0	2
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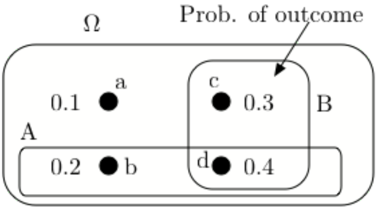
$$cov(X, Y) = E[XY] - E[X]E[Y]$$
$$=$$

Quiz 1: G



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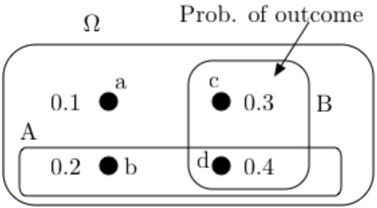
   $= 0.8 - 0.6 \times 1.4 =$

Quiz 1: G



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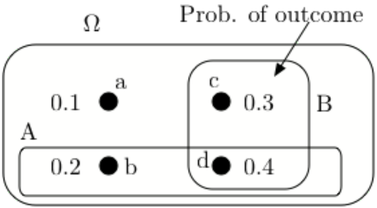

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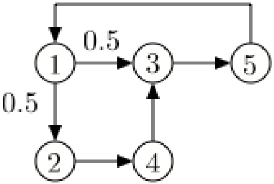
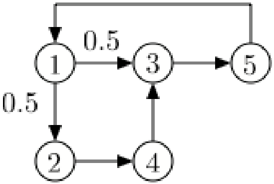
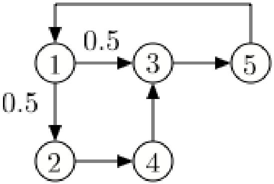


Quiz 1: G



7. Is this Markov chains irreducible?

Quiz 1: G



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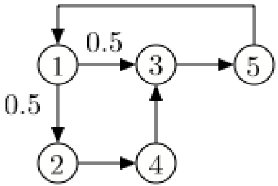

7. Is this Markov chains irreducible? Yes.
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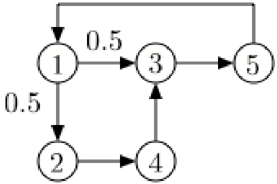
Quiz 1: G



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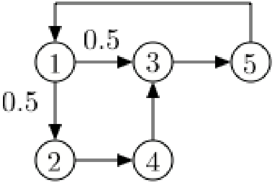
   No.

Quiz 1: G



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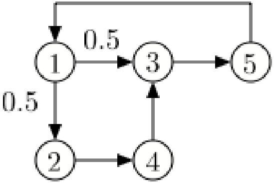
Quiz 1: G



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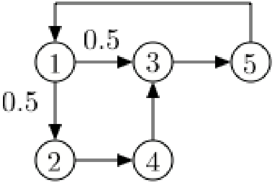
Quiz 1: G



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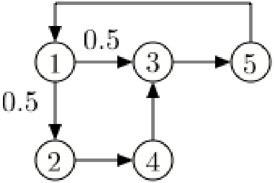
Quiz 1: G



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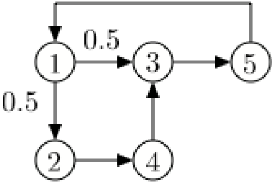
Quiz 1: G



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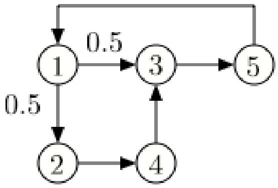

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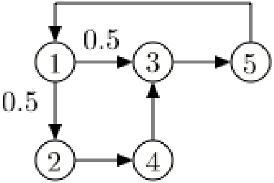
10. Does $\frac{1}{n}\sum_{m=1}^{n-1} 1\{X_m = 3\}$ converge as $n \to \infty$?

Quiz 1: G



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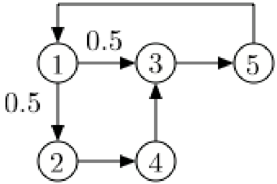

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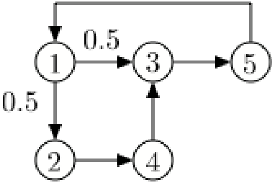
11. Calculate π.

Quiz 1: G



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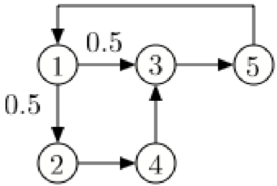
    Let $a = \pi(1)$.

Quiz 1: G



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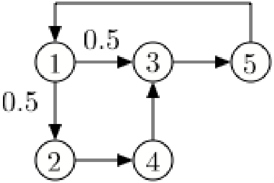
11. Calculate π.

    Let $a = \pi(1)$. Then $a = \pi(5)$,

Quiz 1: G



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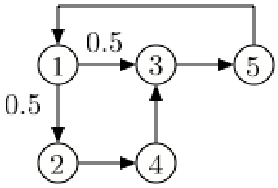
11. Calculate π.

    Let $a = \pi(1)$. Then $a = \pi(5), \pi(2) = 0.5a$,

Quiz 1: G



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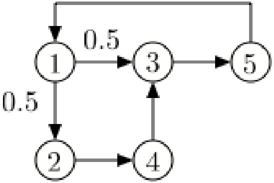
11. Calculate π.

    Let $a = \pi(1)$. Then $a = \pi(5), \pi(2) = 0.5a$,
    $\pi(4) = \pi(2) = 0.5a$,

Quiz 1: G



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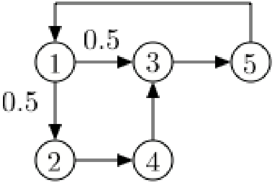
Quiz 1: G



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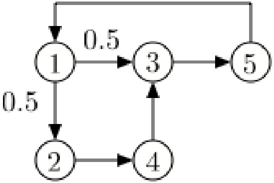
    Thus, $\pi = [a, 0.5a, a, 0.5a, a] = [1, 0.5, 1, 0.5, 1]a$, so $a =$

Quiz 1: G



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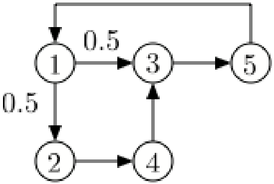
    Let $a = \pi(1)$. Then $a = \pi(5), \pi(2) = 0.5a$,
    $\pi(4) = \pi(2) = 0.5a, \pi(3) = 0.5\pi(1) + \pi(4) = a$.
    Thus, $\pi = [a, 0.5a, a, 0.5a, a] = [1, 0.5, 1, 0.5, 1]a$, so $a = 1/4$.

Quiz 1: G

Quiz 1: G



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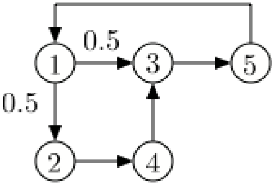
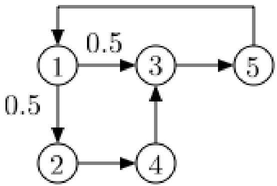


12. Write the first step equations for calculating the mean time from 1 to 4.

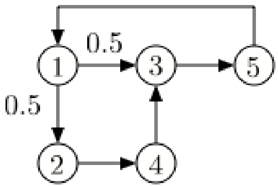
Quiz 1: G



12. Write the first step equations for calculating the mean time from 1 to 4.

$$\beta(1) = 1 + 0.5\beta(2) + 0.5\beta(3)$$

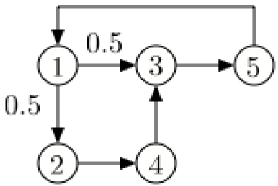
Quiz 1: G



12. Write the first step equations for calculating the mean time from 1 to 4.

$$\beta(1) = 1 + 0.5\beta(2) + 0.5\beta(3)$$
$$\beta(2) = 1$$

Quiz 1: G



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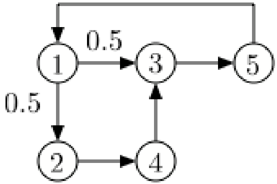
$$\beta(2) = 1$$
$$\beta(3) = 1 + \beta(5)$$

Quiz 1: G



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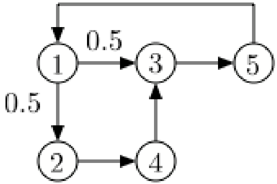
Quiz 1: G



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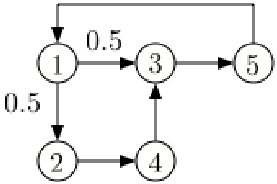
13. Solve these equations.

Quiz 1: G



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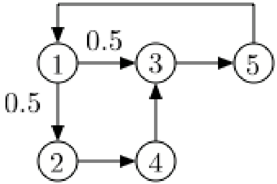
$$\beta(5) = 1 + \beta(1).$$

13. Solve these equations.

$$\beta(1) \quad = \quad 1 + 0.5 \times 1 + 0.5 \times (1 + (1 + \beta(1)))$$

Quiz 1: G



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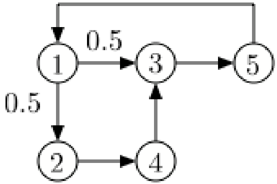
$$\begin{aligned} \beta(1) &= 1 + 0.5 \times 1 + 0.5 \times (1 + (1 + \beta(1))) \\ &= 2.5 + 0.5\beta(1). \end{aligned}$$

Quiz 1: G



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13. Solve these equations.

$$\begin{aligned} \beta(1) &= 1 + 0.5 \times 1 + 0.5 \times (1 + (1 + \beta(1))) \\ &= 2.5 + 0.5\beta(1). \end{aligned}$$

Hence, $\beta(1) = 5$.

Quiz 1: G

14. Which is $E[Y|X]$? Blue, red or green?

Quiz 1: G

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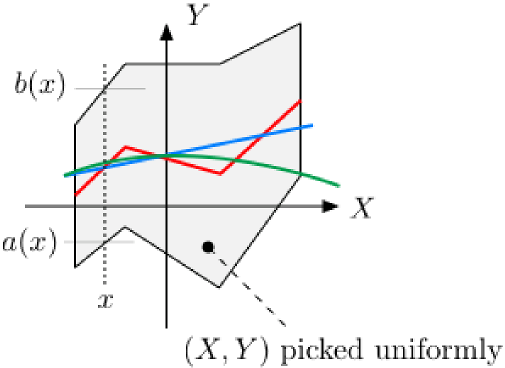

Quiz 1: G

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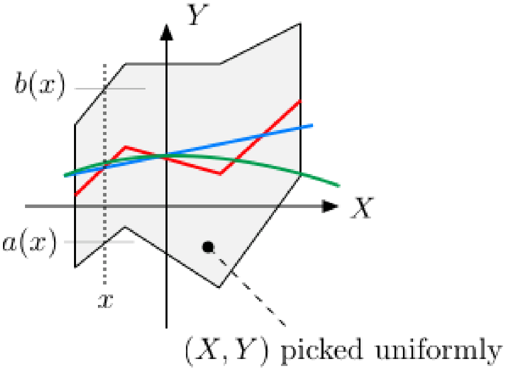

Answer: Red.

Quiz 1: G

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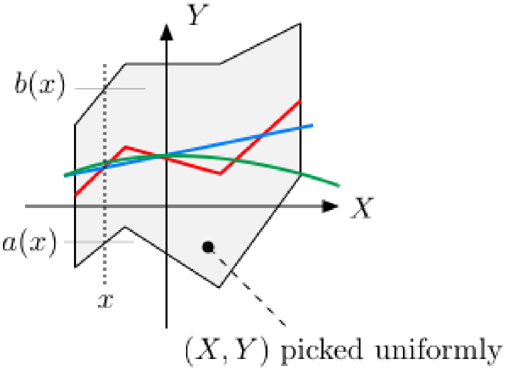
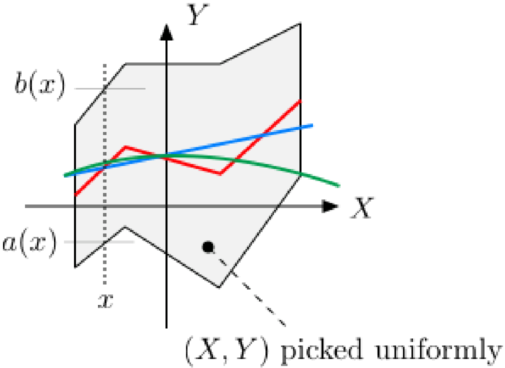


Answer: Red.
Given $X = x$, $Y = U[a(x), b(x)]$.

Quiz 1: G

14. Which is $E[Y|X]$? Blue, red or green?



Answer: Red.
Given $X = x$, $Y = U[a(x), b(x)]$. Thus, $E[Y|X = x] = \frac{a(x)+b(x)}{2}$.

Quiz 2: PG

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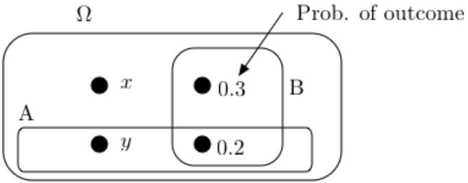
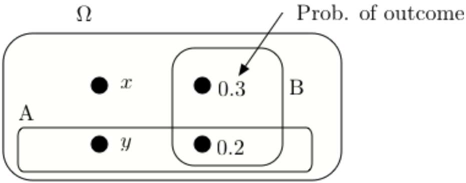
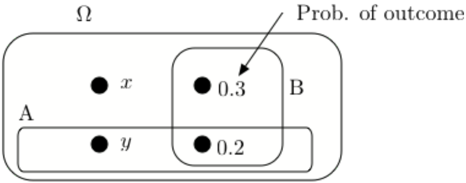


Quiz 2: PG



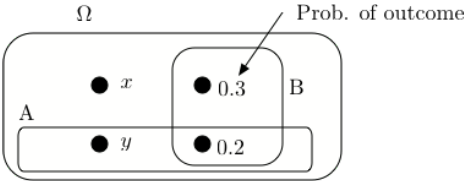
1. Find (x, y) so that A and B are independent.

Quiz 2: PG



1. Find (x, y) so that A and B are independent.
   We need
   $$Pr[A \cap B] = Pr[A]Pr[B]$$

Quiz 2: PG



1. Find (x, y) so that A and B are independent.

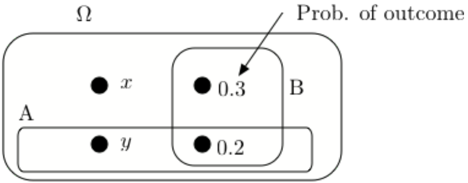
   We need

   $$Pr[A \cap B] = Pr[A]Pr[B]$$

   That is,

   $$0.2 = (y + 0.2) \times 0.5 =$$

Quiz 2: PG



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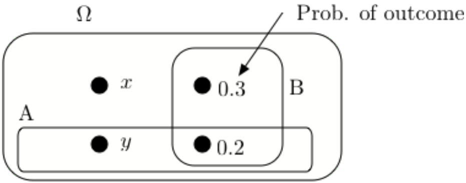

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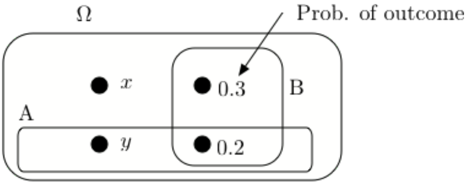

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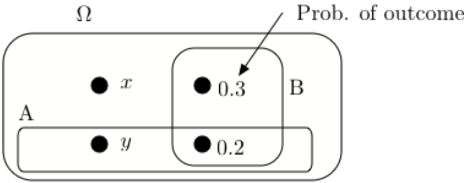

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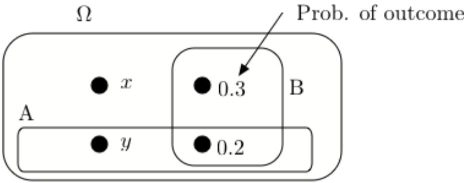

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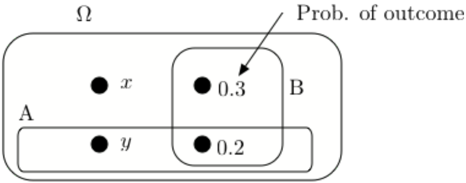

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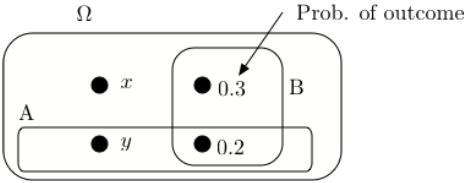

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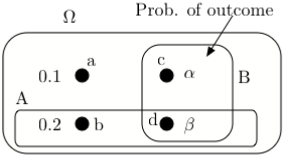

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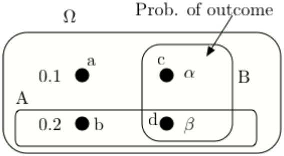

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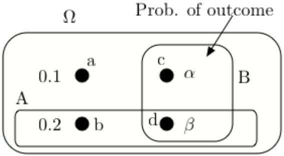

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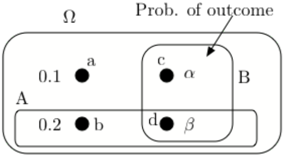

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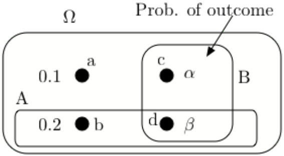

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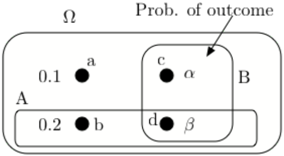

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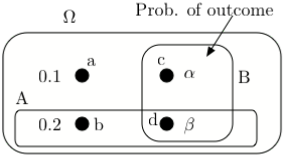

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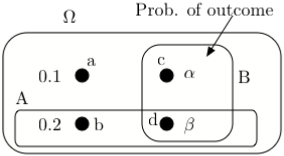

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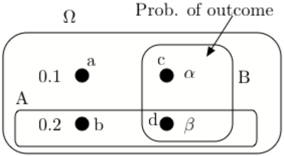

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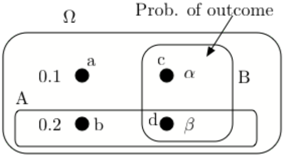

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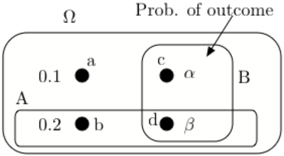

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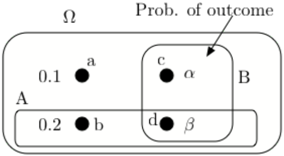

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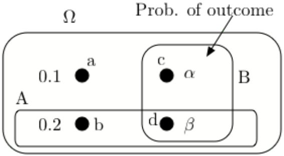

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Quiz 2: PG

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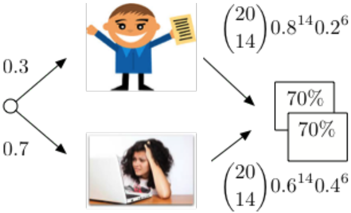

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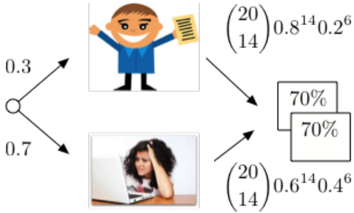

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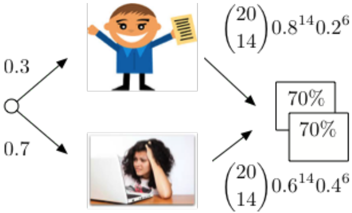

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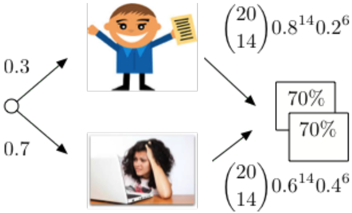

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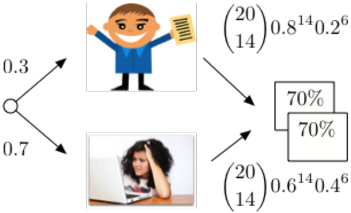

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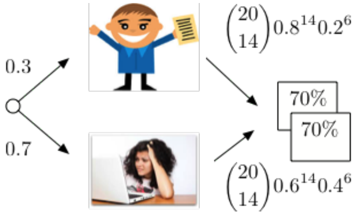

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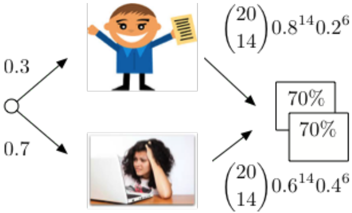

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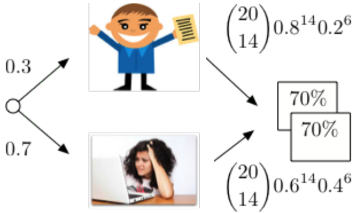

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6. You roll a balanced six-sided die 20 times. Use CLT to upper-bound the probability that the total number of dots exceeds 85.

Let $X = X_1 + \cdots + X_{20}$ be the total number of dots.
Then

$$\frac{X - 70}{\sigma\sqrt{20}} \approx \mathcal{N}(0,1)$$

where

$$\sigma^2 = var(X_1) = (1/6)\sum_{m=1}^{6} m^2 - (3.5)^2 \approx 2.9 = 1.7^2.$$

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$$\begin{aligned} Pr[X > 85] &= Pr[X - 70 > 15] \\ &= Pr[\frac{X - 70}{1.7 \times 4.5} > \frac{15}{1.7 \times 4.5}] \end{aligned}$$

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Quiz 2: PG

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Hence,

$$Pr[X > 85] \leq \frac{58}{15^2} \approx 0.26.$$

Quiz 3: R

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Quiz 3: R

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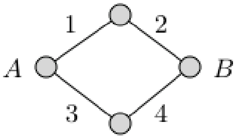
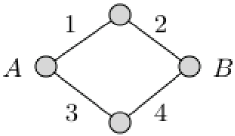


Quiz 3: R



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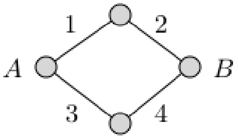
Quiz 3: R



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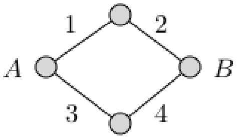

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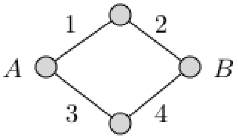

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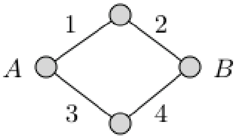

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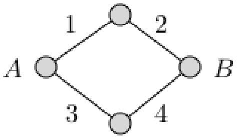

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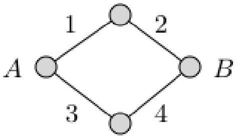

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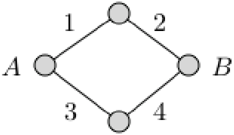

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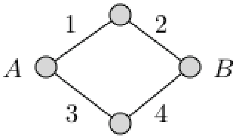

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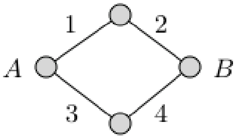

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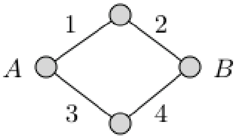

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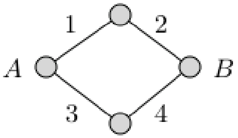

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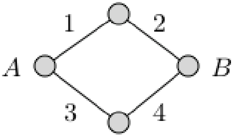

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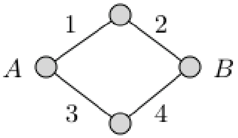
We are looking for $E[Z]$ where $Z = \max\{Y_1, Y_2\}$ with $Y_1 = \min\{X_1, X_2\}$ and $Y_2 = \min\{X_3, X_4\}$.

$$\begin{aligned}
Pr[Y_1 > t] &= Pr[X_1 > t]Pr[X_2 > t] = (1-t)^2 \\
Pr[Z \leq t] &= Pr[Y_1 \leq t]Pr[Y_2 \leq t] = (1-(1-t)^2)^2 \\
&= (2t - t^2)^2 = 4t^2 - 4t^3 + t^4 \\
f_Z(t) &= 8t - 12t^2 + 4t^3 \\
E[Z] &= \int_0^1 t f_Z(t) dt
\end{aligned}$$

Quiz 3: R



2. In the figure, $1, 2, 3, 4$ are links that fail after i.i.d. times that are $U[0, 1]$.

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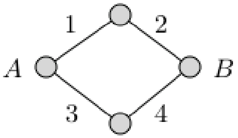

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Quiz 3: R

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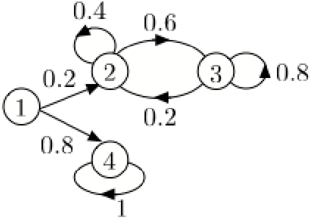

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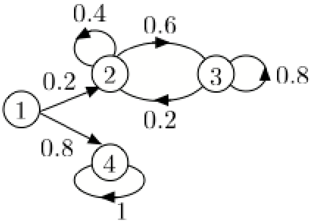

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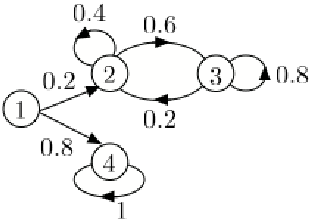
Quiz 3: R



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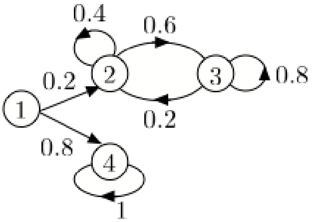

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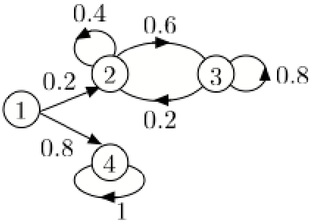

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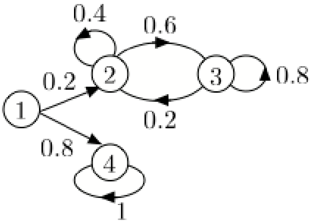

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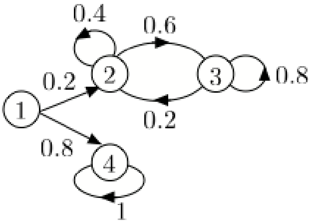

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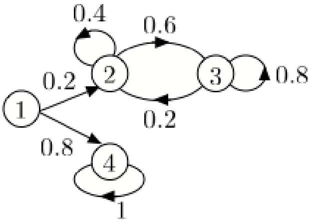

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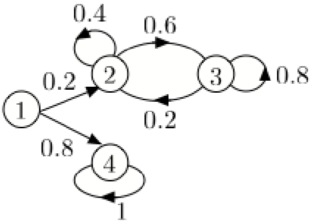

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- $3 \times 3.5 = 12.5$. No.
- $E[X^2] = E[X]^2$. No.
- $X = B(n,p) \implies var(X) = n^2 p(1-p)$. No.
- $Pr[X = a] = Pr[X = a|A] + Pr[X = a|\bar{A}]$. No.
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Common Mistakes

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