Lecture 6.

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Euler's Formula.

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Planar Six and then Five Color theorem.

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Types of graphs.

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Types of graphs.

- Complete Graphs.
- Trees (a little more.)
- Hypercubes.

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Planar graphs.

A graph that can be drawn in the plane without edge crossings.

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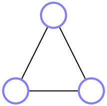

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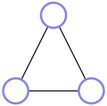

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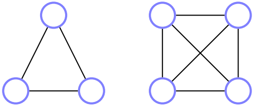

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Four node complete?

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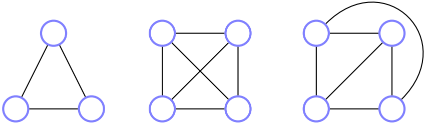

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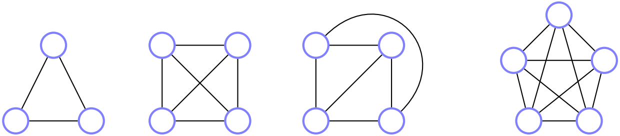

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Five node complete or K_5 ?

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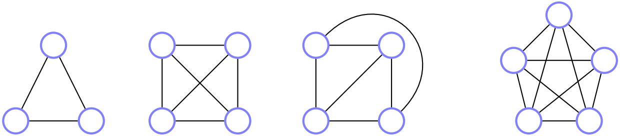

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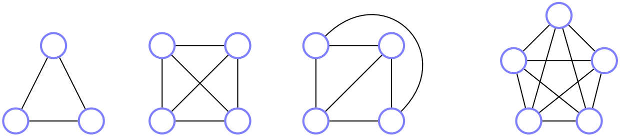

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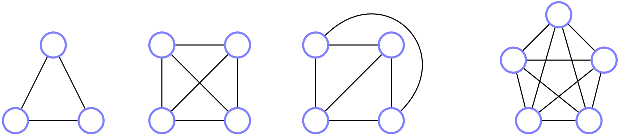

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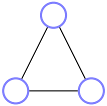 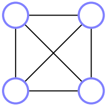 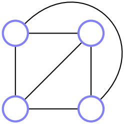 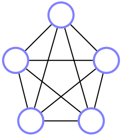

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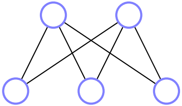

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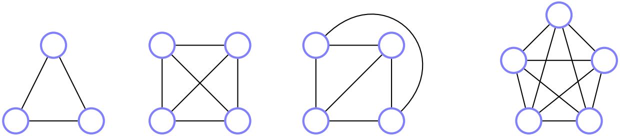

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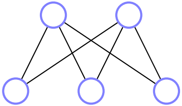

Two to three nodes, bipartite?

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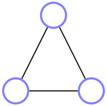
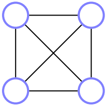
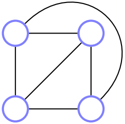
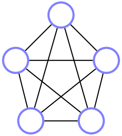

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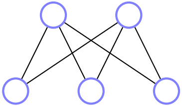
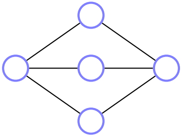

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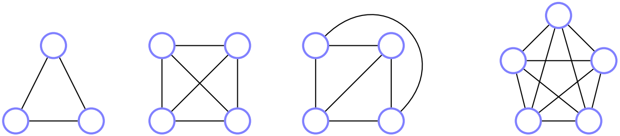

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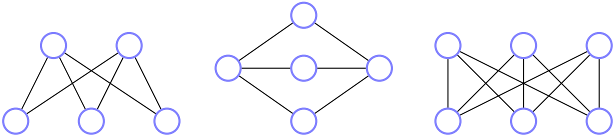

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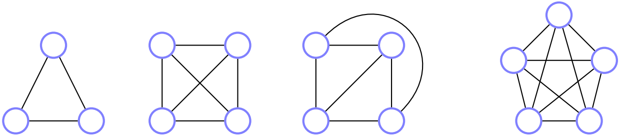

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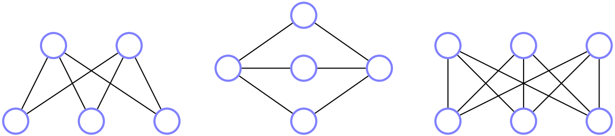

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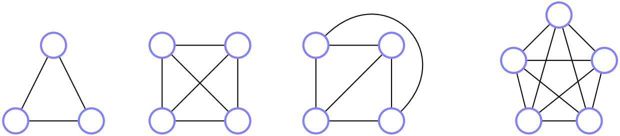

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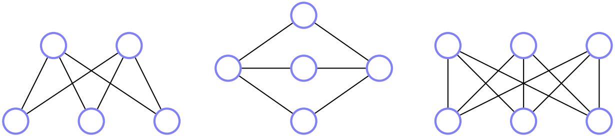

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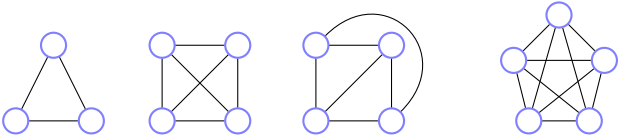

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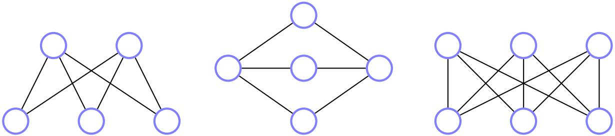

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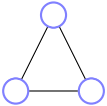
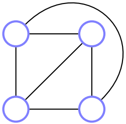
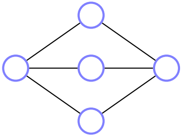
Euler's Formula.

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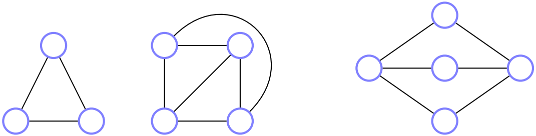
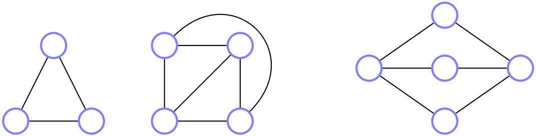


Faces: connected regions of the plane.

Euler's Formula.



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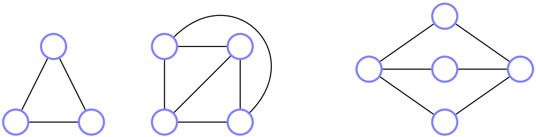

Faces: connected regions of the plane.

How many faces for
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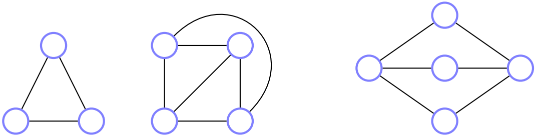

Faces: connected regions of the plane.

How many faces for

triangle? 2

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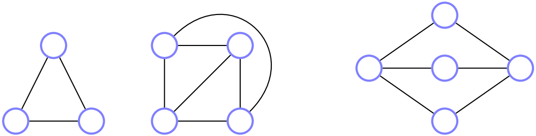

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How many faces for
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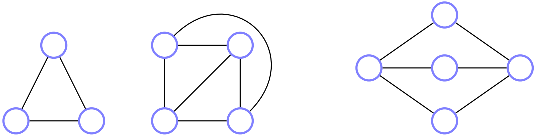

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How many faces for

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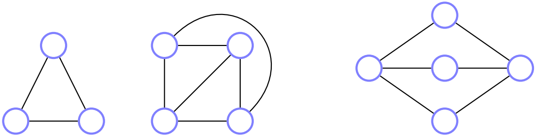

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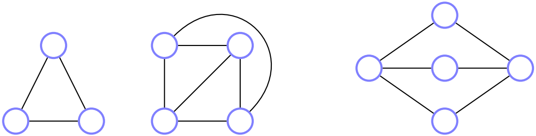

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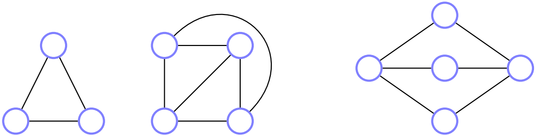

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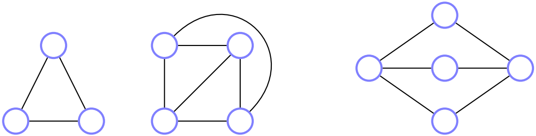

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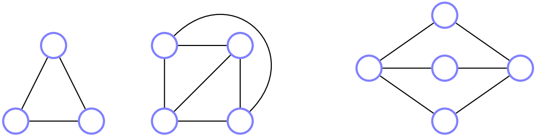

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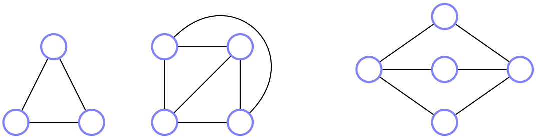

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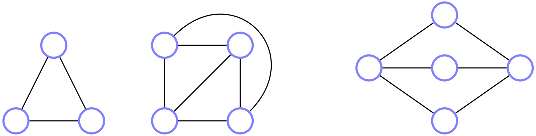

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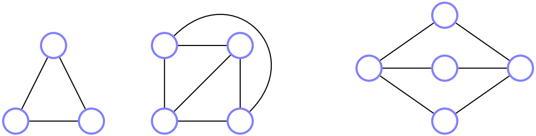

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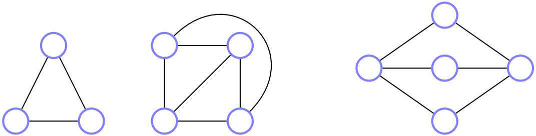

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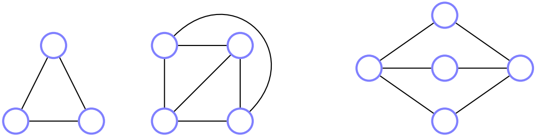

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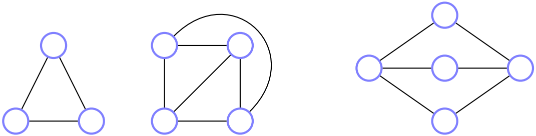

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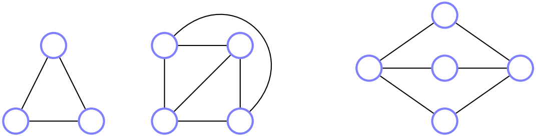

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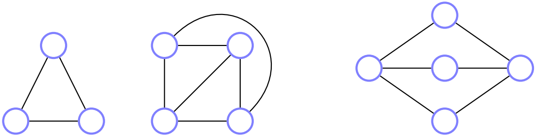

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Euler's Formula: Connected planar graph has $v + f = e + 2$.

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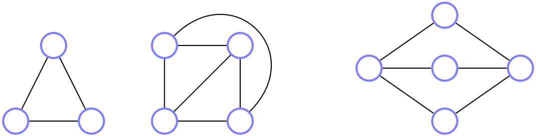

Faces: connected regions of the plane.

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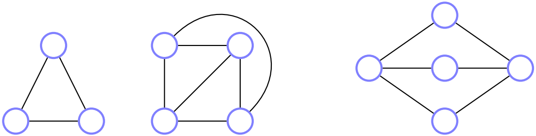

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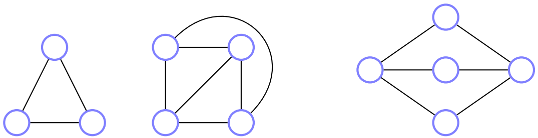

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Euler and Polyhedron.

Greeks knew formula for polyhedron.

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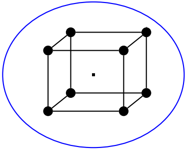

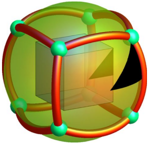

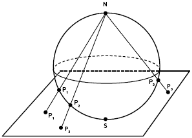

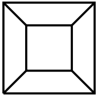

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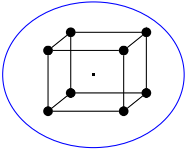

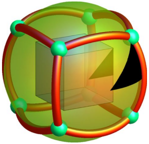

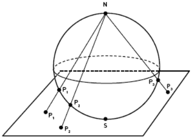

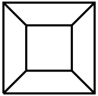

Faces?

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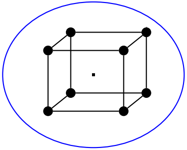

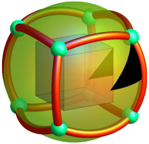

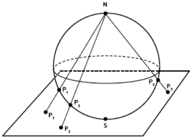

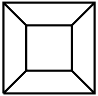

Faces? 6. Edges?

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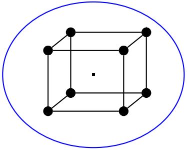

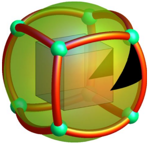

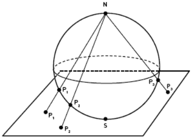

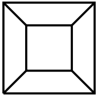

Faces? 6. Edges? 12.

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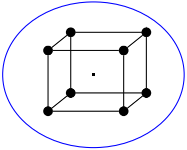 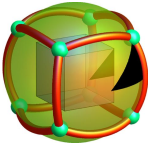 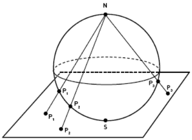 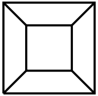

Faces? 6.   Edges? 12.   Vertices?

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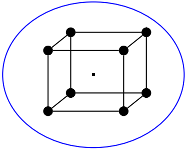
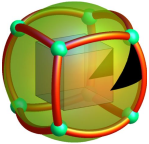
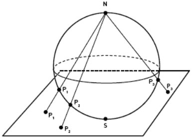

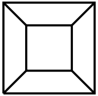

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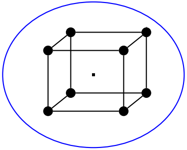
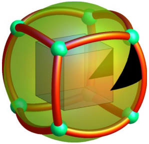
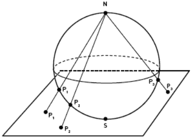

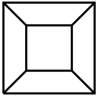

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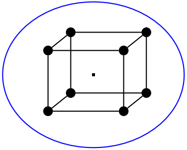

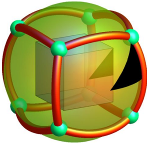

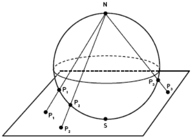

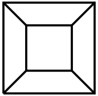

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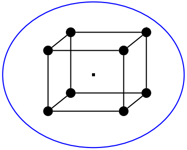

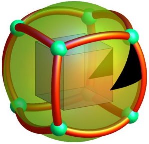

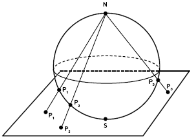

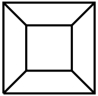

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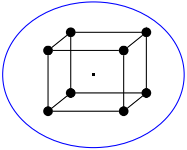
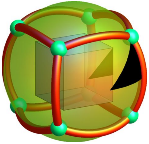
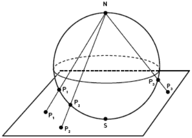

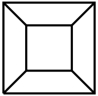

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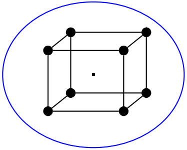
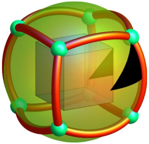
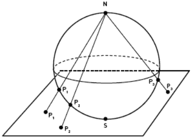

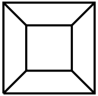

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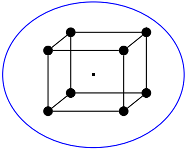
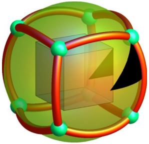
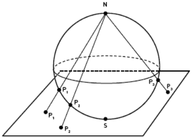

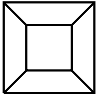

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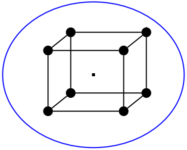

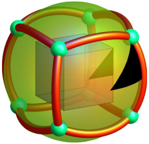

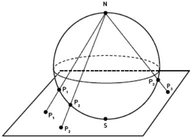

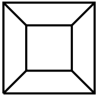

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Polyhedron without holes

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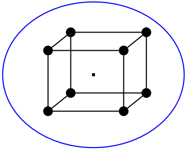
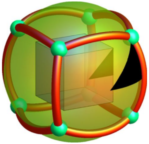
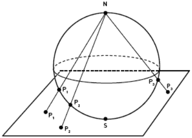

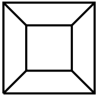

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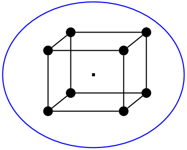
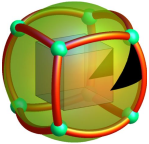
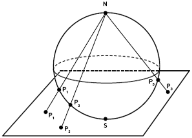

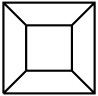

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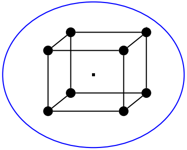
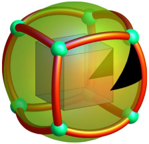
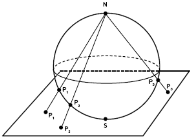

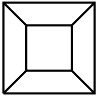

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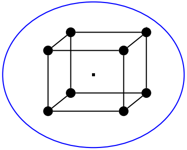
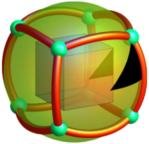
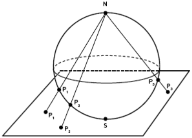

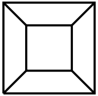

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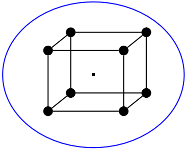
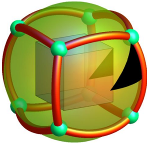
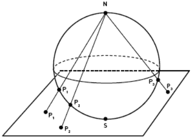

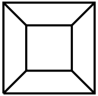

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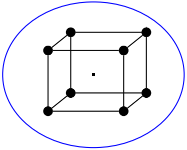

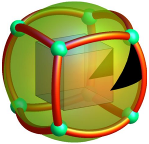

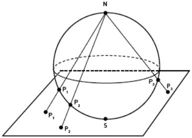

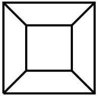

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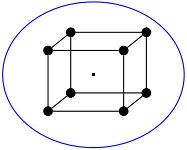
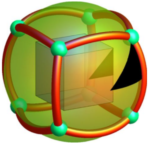
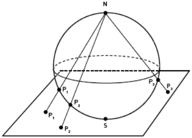

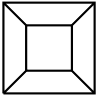

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Project Sphere-N

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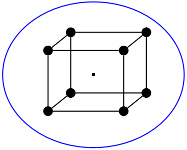
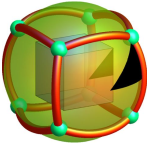
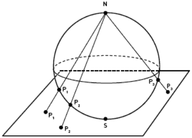

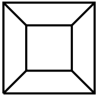

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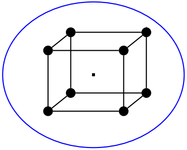
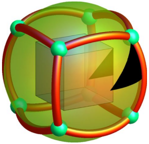
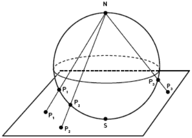

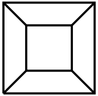

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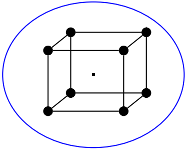

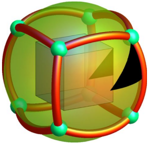

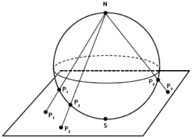

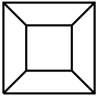

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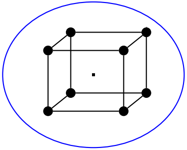

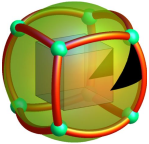

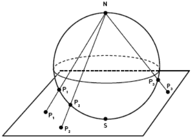

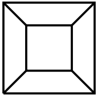

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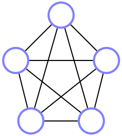
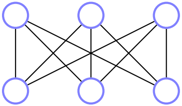
Project Sphere-N onto Plane: drawing on plane.

Euler proved formula thousands of years later!

Euler and non-planarity of K_5 and $K_{3,3}$





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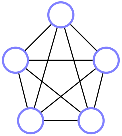

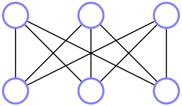

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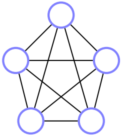

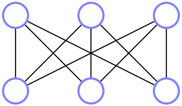

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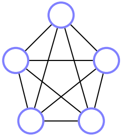

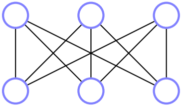

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Consider Face edge Adjacencies.

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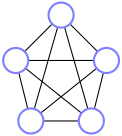

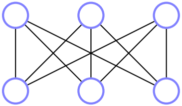

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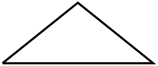

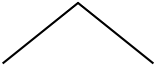

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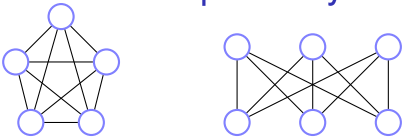

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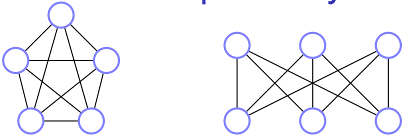
Each face is adjacent to at least three edges.

Euler and non-planarity of K_5 and $K_{3,3}$



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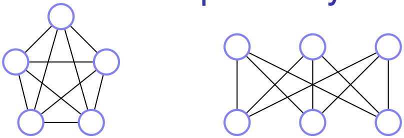

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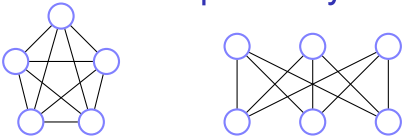

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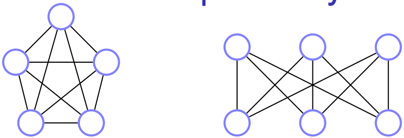

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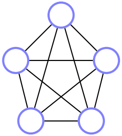

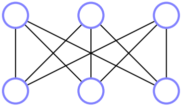

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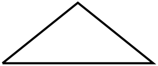

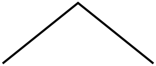

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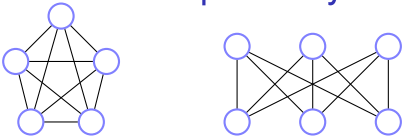
Euler and non-planarity of K_5 and $K_{3,3}$



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We consider simple graphs where $v \geq 3$.

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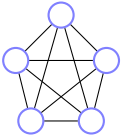

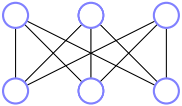

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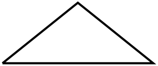

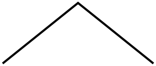

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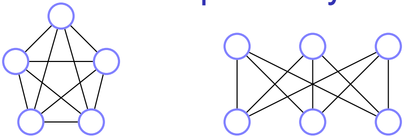

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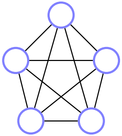

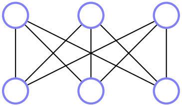

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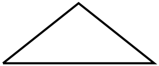

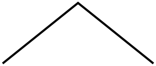

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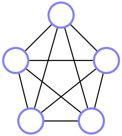
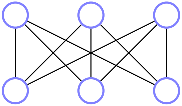

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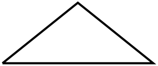
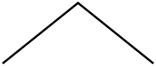

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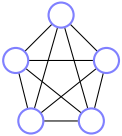

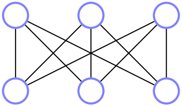

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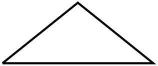

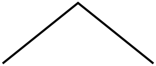

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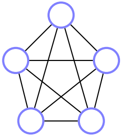

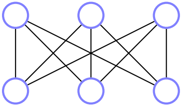

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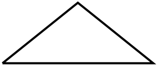

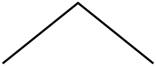

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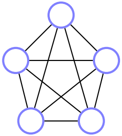

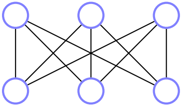

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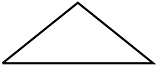

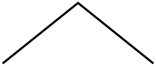

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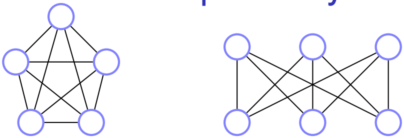
K_5

Euler and non-planarity of K_5 and $K_{3,3}$



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K_5 Edges?

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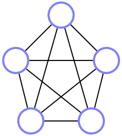

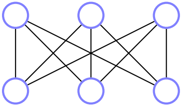

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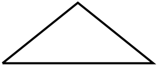

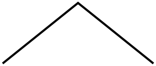

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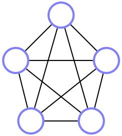

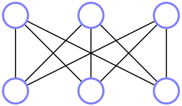

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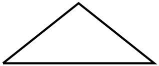

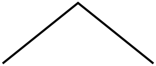

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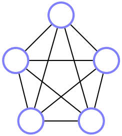
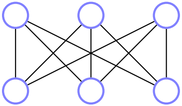
Euler and non-planarity of K_5 and $K_{3,3}$





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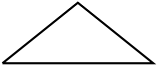

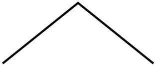

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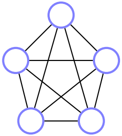

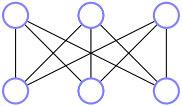

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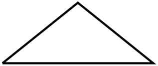

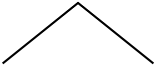

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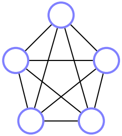

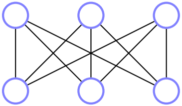

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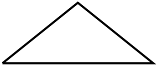

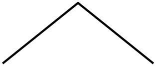

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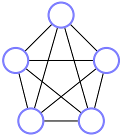

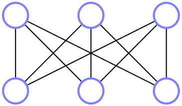

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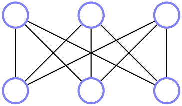
Proving non-planarity for $K_{3,3}$



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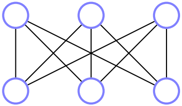
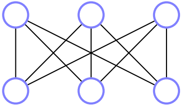


$K_{3,3}$?

Proving non-planarity for $K_{3,3}$



$K_{3,3}$? Edges?

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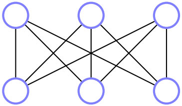
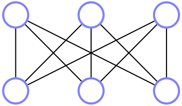


$K_{3,3}$? Edges? 9.

Proving non-planarity for $K_{3,3}$



$K_{3,3}$? Edges? 9. Vertices. 6.

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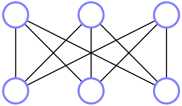

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$e \leq 3(v) - 6$ for planar graphs.

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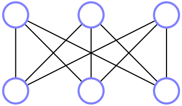

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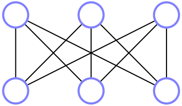

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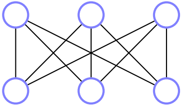

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Step in proof of K_5: faces are adjacent to ≥ 3 edges.

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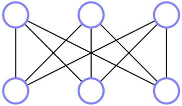

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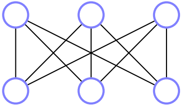
For $K_{3,3}$ every cycle is of even length or ≥ 4.

Proving non-planarity for $K_{3,3}$



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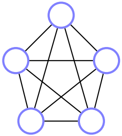
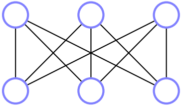
Finish in homework!

Planarity and Euler





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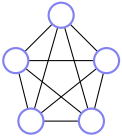

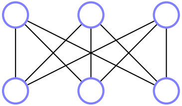

These graphs **cannot** be drawn in the plane without edge crossings.

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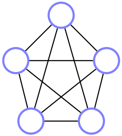

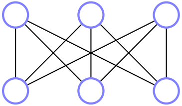

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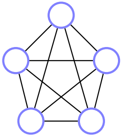
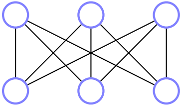
Euler's Formula: $v + f = e + 2$ for any planar drawing.

Planarity and Euler





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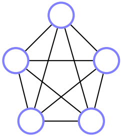

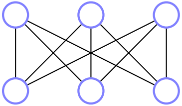

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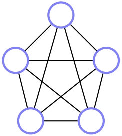

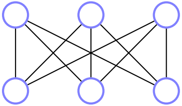

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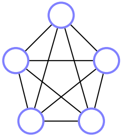

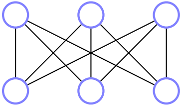

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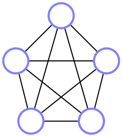

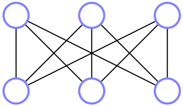

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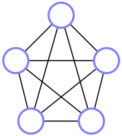

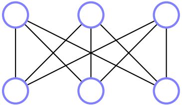

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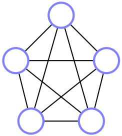

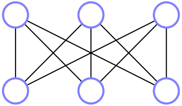

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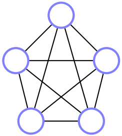

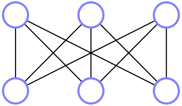

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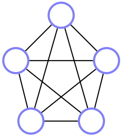

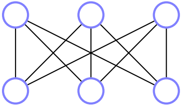

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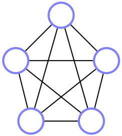
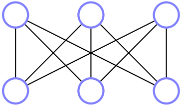
Proved absolutely no drawing can work for these graphs.

Planarity and Euler

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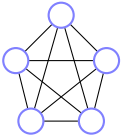
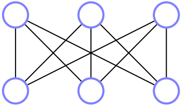
So...

Planarity and Euler





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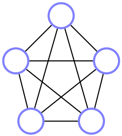
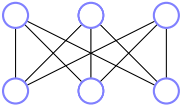
Planarity and Euler





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So......so ...Cool!

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Euler's formula.

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Proof:

Euler's formula.

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Proof: Induction on e.

Euler's formula.

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Proof: Induction on e.
Base:

Euler's formula.

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Proof: Induction on *e*.
Base: $e=0$,

Euler's formula.

Euler: Connected planar graph has $v+f=e+2$.

Proof: Induction on e.
Base: $e=0$, $v=f=1$.

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof: Induction on e.
Base: $e = 0$, $v = f = 1$.
Induction Step:

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof: Induction on e.
Base: $e = 0$, $v = f = 1$.
Induction Step:
If it is a tree.

Euler's formula.

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Proof: Induction on e.
Base: $e = 0$, $v = f = 1$.
Induction Step:
If it is a tree. $e = v - 1$, $f = 1$, $v + 1 = (v - 1) + 2$. Yes.

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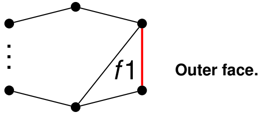

Joins two faces.

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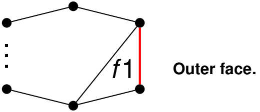

Joins two faces.
New graph: v-vertices.

Euler's formula.

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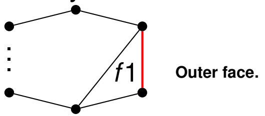

Joins two faces.
New graph: v-vertices. $e-1$ edges.

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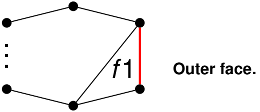

Joins two faces.
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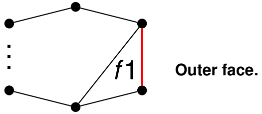

Joins two faces.
New graph: v-vertices. $e - 1$ edges. $f - 1$ faces. Planar.

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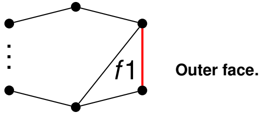
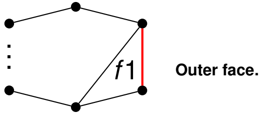


Joins two faces.
New graph: v-vertices. $e-1$ edges. $f-1$ faces. Planar.
$v+(f-1)=(e-1)+2$ by induction hypothesis.

Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

Proof: Induction on e.
Base: $e = 0$, $v = f = 1$.
Induction Step:
If it is a tree. $e = v - 1$, $f = 1$, $v + 1 = (v - 1) + 2$. Yes.
If not a tree.
Find a cycle. Remove edge.



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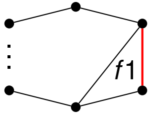

Outer face.

Joins two faces.
New graph: v-vertices. $e - 1$ edges. $f - 1$ faces. Planar.
$v + (f - 1) = (e - 1) + 2$ by induction hypothesis.
Therefore $v + f = e + 2$. □

Quick:
$v + 1 = (v - 1) + 2$, add edge: $f \to f + 1$, $e \to e + 1$.

Graph Coloring.

Given $G = (V, E)$, a coloring of G assigns colors to vertices V where for each edge the endpoints have different colors.

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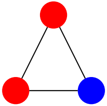

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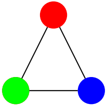

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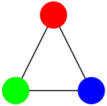

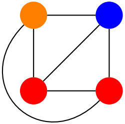

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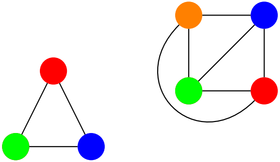

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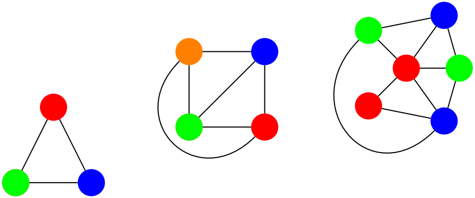

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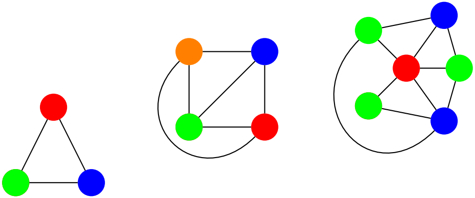

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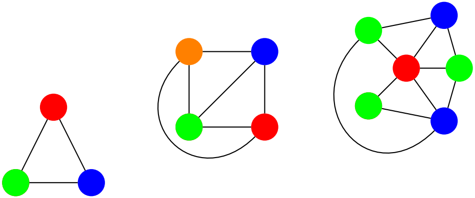

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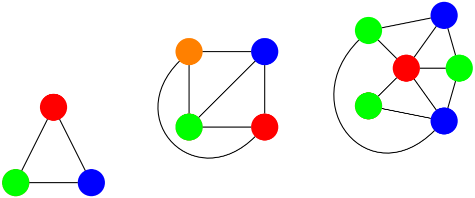

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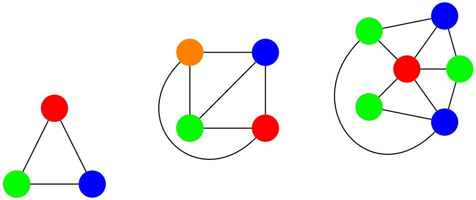

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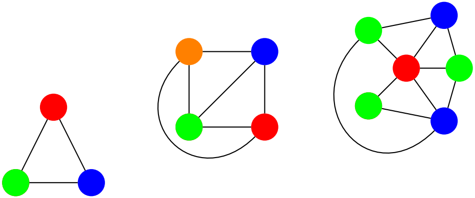

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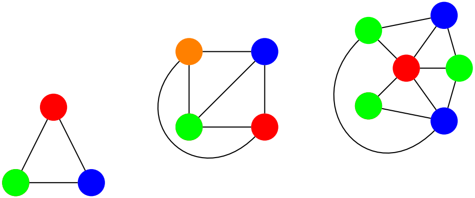

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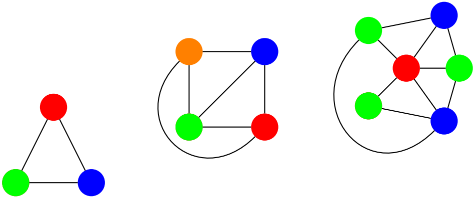

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Interesting things to do.

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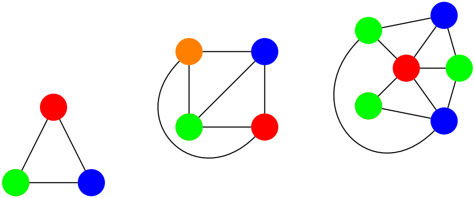

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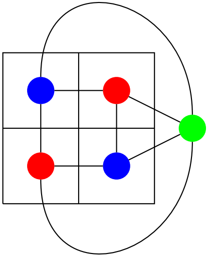
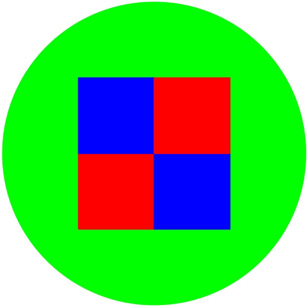
Interesting things to do. Algorithm!

Planar graphs and maps.

Planar graph coloring $\equiv$ map coloring.





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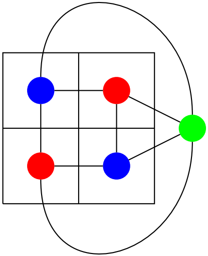

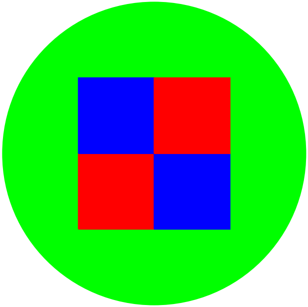

Four color theorem is about planar graphs!

Six color theorem.

Theorem: Every planar graph can be colored with six colors.

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Recall: $e \leq 3v - 6$ for any planar graph where $v > 2$.

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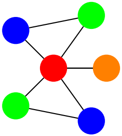
Five color theorem: prelimnary.

Preliminary Observation: Connected components of vertices with two colors in a legal coloring can switch colors.



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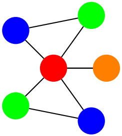

Look at only green and blue.

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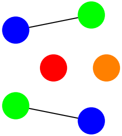

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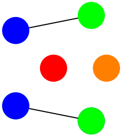

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Can switch in one component.

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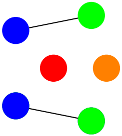

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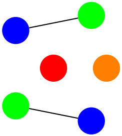

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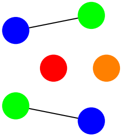

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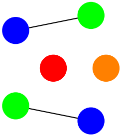

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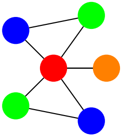

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Proof: Again with the degree 5 vertex.

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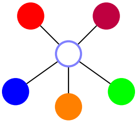

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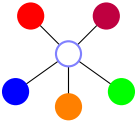

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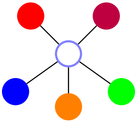

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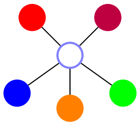

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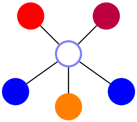

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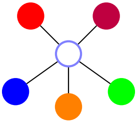

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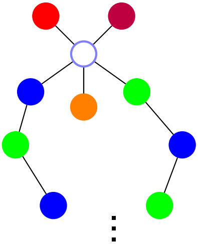

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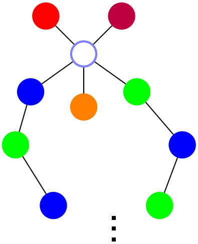

Assume neighbors are colored all differently.
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Switch green and blue in green's component.
Done. Unless blue-green path to blue.
Switch orange and red in oranges component.

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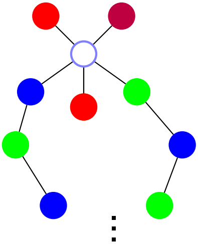

Assume neighbors are colored all differently.
Otherwise one of 5 colors is available. $\Longrightarrow$ Done!
Switch green and blue in green's component.
Done. Unless blue-green path to blue.
Switch orange and red in oranges component.
Done.

Five color theorem

Theorem: Every planar graph can be colored with five colors.

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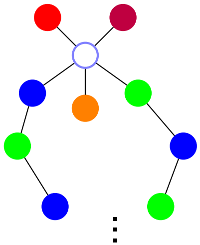

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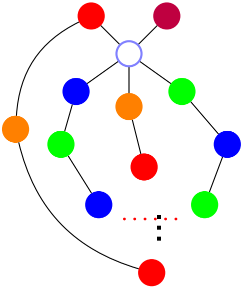

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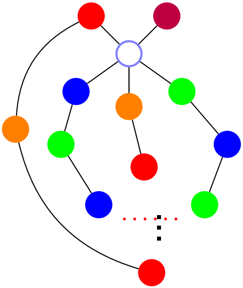

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Planar.

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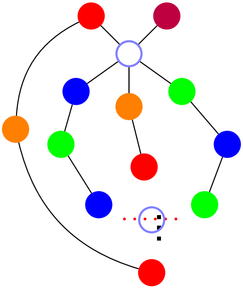
Proof: Again with the degree 5 vertex. Again recurse.



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Planar. $\implies$ paths intersect at a vertex!

Five color theorem

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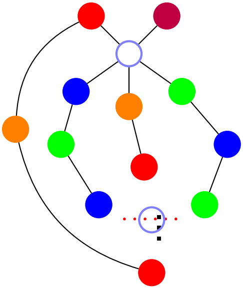

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What color is it?

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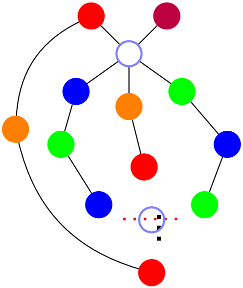

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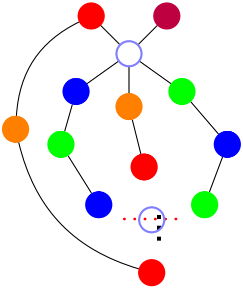

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What color is it?
Must be blue or green to be on that path.

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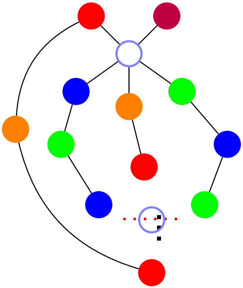

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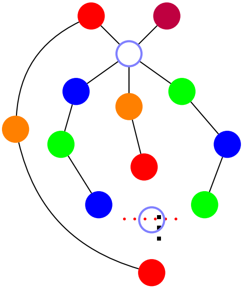

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Planar. $\implies$ paths intersect at a vertex!

What color is it?
Must be blue or green to be on that path.
Must be red or orange to be on that path.

Contradiction.

Five color theorem

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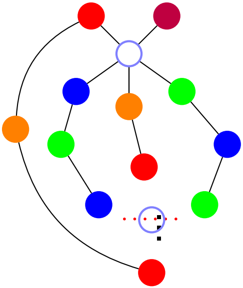
Proof: Again with the degree 5 vertex. Again recurse.



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Planar. $\implies$ paths intersect at a vertex!

What color is it?
Must be blue or green to be on that path.
Must be red or orange to be on that path.

Contradiction. Can recolor one of the neighbors.

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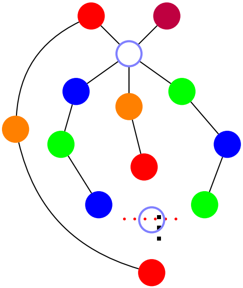

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Done. Unless red-orange path to red.

Planar. $\implies$ paths intersect at a vertex!

What color is it?
Must be blue or green to be on that path.
Must be red or orange to be on that path.

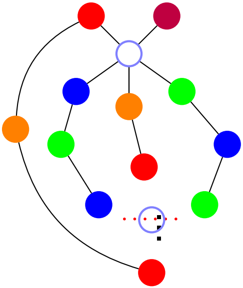
Contradiction. Can recolor one of the neighbors.
Gives an available color for center vertex!

Five color theorem

Theorem: Every planar graph can be colored with five colors.

Preliminary Observation: Connected components of vertices with two colors in a legal coloring can switch colors.

Proof: Again with the degree 5 vertex. Again recurse.



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Planar. $\implies$ paths intersect at a vertex!

What color is it?

Must be blue or green to be on that path.

Must be red or orange to be on that path.

Contradiction. Can recolor one of the neighbors.

Gives an available color for center vertex! □

Four Color Theorem

Four Color Theorem

Theorem: Any planar graph can be colored with four colors.

Four Color Theorem

Theorem: Any planar graph can be colored with four colors.

Proof:

Four Color Theorem

Theorem: Any planar graph can be colored with four colors.

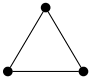
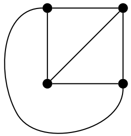
Proof: Not Today!

Four Color Theorem

Theorem: Any planar graph can be colored with four colors.

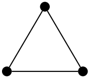
Proof: Not Today!

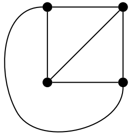
Complete Graph.

K_n complete graph on n vertices.

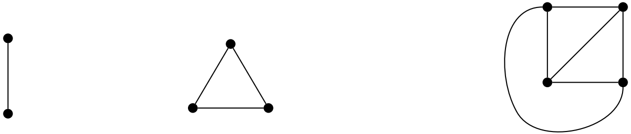
Complete Graph.





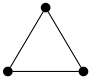
K_n complete graph on n vertices.
All edges are present.

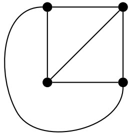
Complete Graph.



K_n complete graph on n vertices.
  All edges are present.
  Everyone is my neighbor.

Complete Graph.





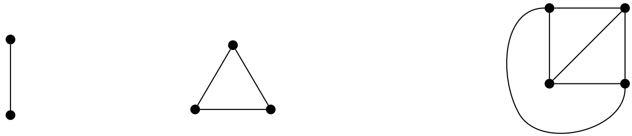
K_n complete graph on n vertices.

All edges are present.

Everyone is my neighbor.

Each vertex is adjacent to every other vertex.

Complete Graph.



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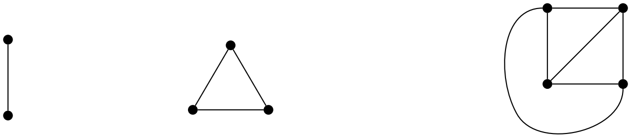

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All edges are present.

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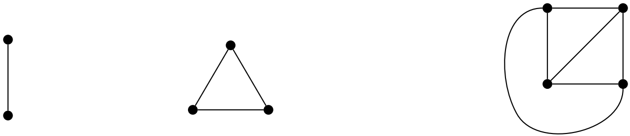
How many edges?

Complete Graph.



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How many edges?

Each vertex is incident to $n-1$ edges.

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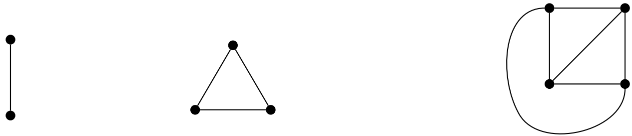

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How many edges?

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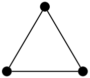
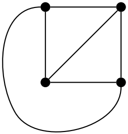
Sum of degrees is $n(n-1)$

Complete Graph.





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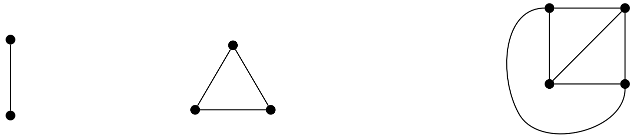
Each vertex is adjacent to every other vertex.

How many edges?

Each vertex is incident to $n-1$ edges.

Sum of degrees is $n(n-1) = 2|E|$

Complete Graph.



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All edges are present.

Everyone is my neighbor.

Each vertex is adjacent to every other vertex.

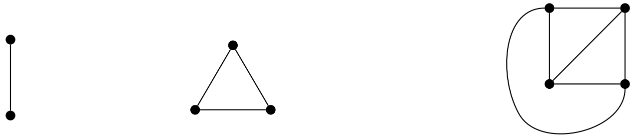
How many edges?

Each vertex is incident to $n-1$ edges.

Sum of degrees is $n(n-1) = 2|E|$

$\implies$ Number of edges is $n(n-1)/2$.

Complete Graph.



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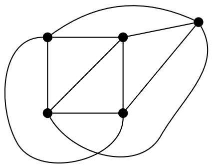
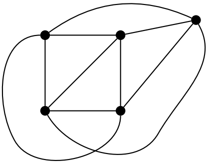
K_4 and K_5



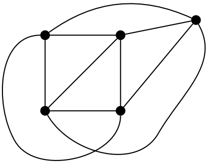
K_5 is not planar.

K_4 and K_5



K_5 is not planar.
Cannot be drawn in the plane without an edge crossing!

K_4 and K_5

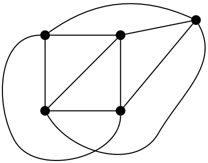


K_5 is not planar.

Cannot be drawn in the plane without an edge crossing!

Prove it!

K_4 and K_5



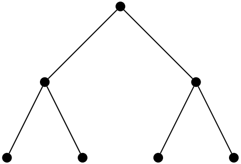
K_5 is not planar.

Cannot be drawn in the plane without an edge crossing!

Prove it! We did!

Tree's fall apart.

Thm: There is one vertex whose removal disconnects $|V|/2$ nodes from each other.



Idea of proof.

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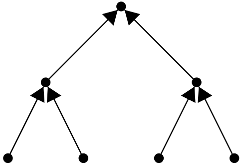
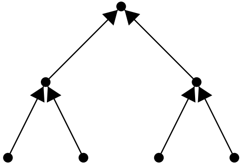
Thm: There is one vertex whose removal disconnects $|V|/2$ nodes from each other.



Idea of proof.
Point edge toward bigger side.

Tree's fall apart.

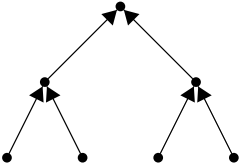
Thm: There is one vertex whose removal disconnects $|V|/2$ nodes from each other.



Idea of proof.
Point edge toward bigger side.
Remove center node:

Tree's fall apart.

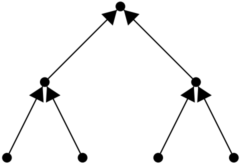
Thm: There is one vertex whose removal disconnects $|V|/2$ nodes from each other.



Idea of proof.
Point edge toward bigger side.
Remove center node: node with no outgoing arc. (Hotel California.)

Tree's fall apart.

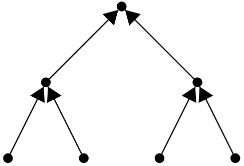
Thm: There is one vertex whose removal disconnects $|V|/2$ nodes from each other.



Idea of proof.
Point edge toward bigger side.
Remove center node: node with no outgoing arc. (Hotel California.)
All the neighbors in components that are smaller than $|V|/2$.

Tree's fall apart.

Thm: There is one vertex whose removal disconnects $|V|/2$ nodes from each other.



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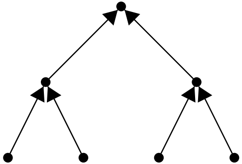

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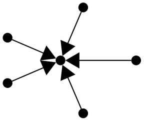

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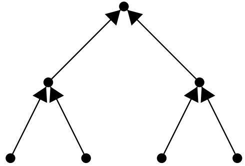

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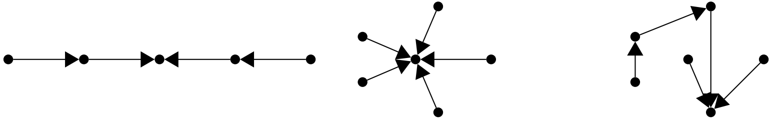

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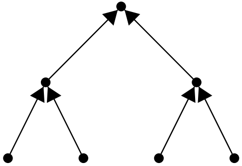

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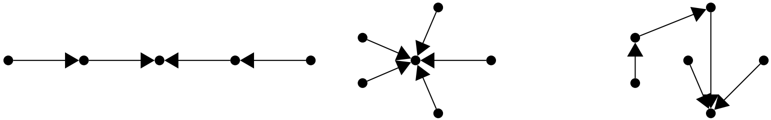

Hypercubes.

Complete graphs, really connected!

Hypercubes.

Complete graphs, really connected! But lots of edges.

$|V|(|V|-1)/2$

Hypercubes.

Complete graphs, really connected! But lots of edges.

$|V|(|V|-1)/2$

Trees,

Hypercubes.

Complete graphs, really connected! But lots of edges.

$|V|(|V|-1)/2$

Trees, few edges. $(|V|-1)$

Hypercubes.

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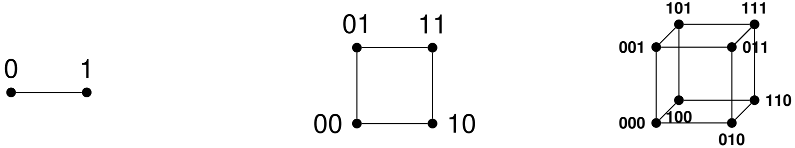

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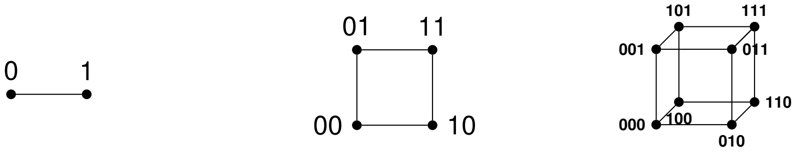

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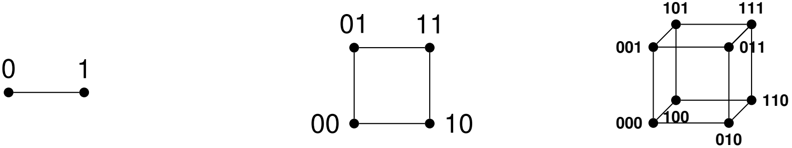

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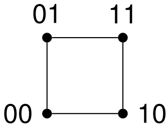

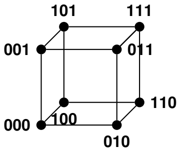

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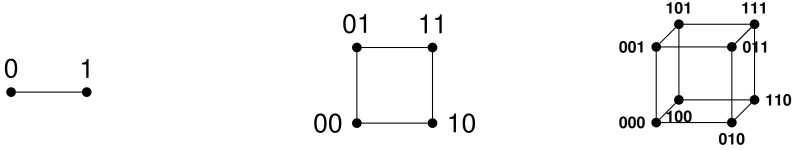

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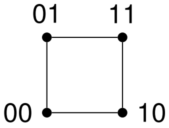

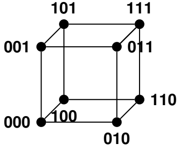

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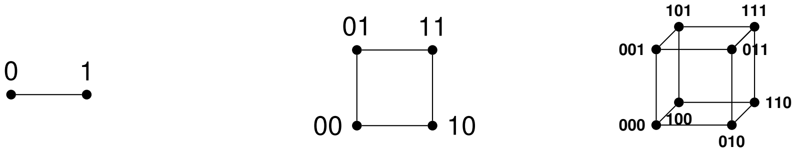

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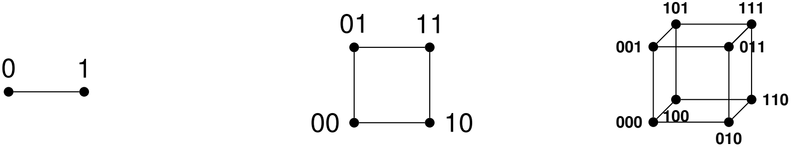

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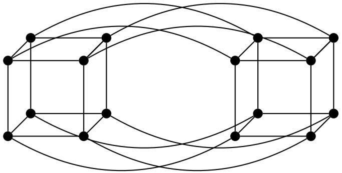

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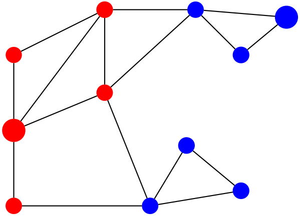
Restatement: for any cut in the hypercube, the number of cut edges is at least the size of the small side.

Cuts in graphs.



S is red, $V - S$ is blue.

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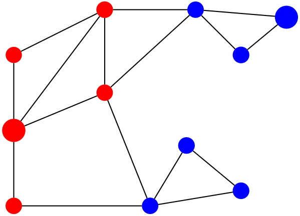

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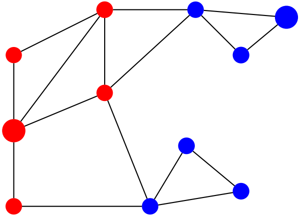

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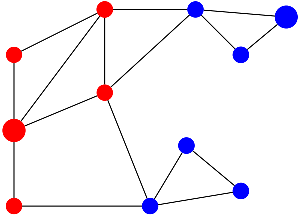

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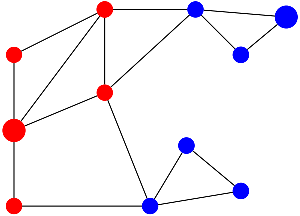

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Hypercube: any cut that cuts off x nodes has $\geq x$ edges.

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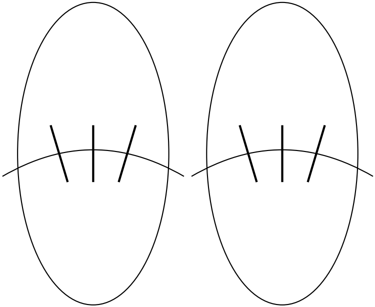

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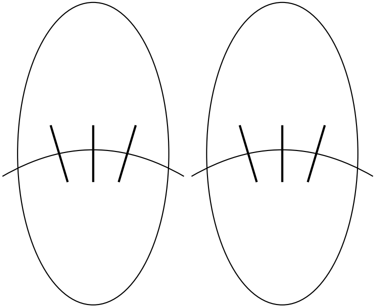

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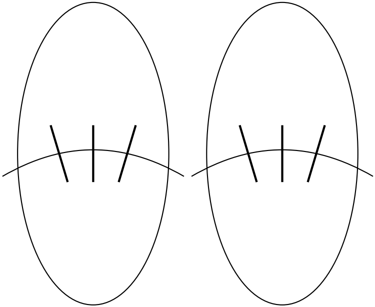

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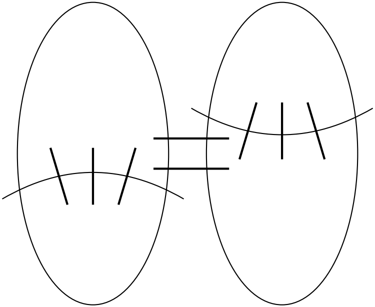

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Edges cut in $H_0 \geq |S_0|$.
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Total cut edges $\geq |S_0| + |S_1| = |S|$.

Induction Step

Thm: For any cut $(S, V - S)$ in the hypercube, the number of cut edges is at least the size of the small side, $|S|$.

Proof: Induction Step.
Recursive definition:
$H_0 = (V_0, E_0)$, $H_1 = (V_1, E_1)$, edges E_x that connect them.
$H = (V_0 \cup V_1, E_0 \cup E_1 \cup E_x)$
$S = S_0 \cup S_1$ where S_0 in first, and S_1 in other.

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Induction Step. Case 2.

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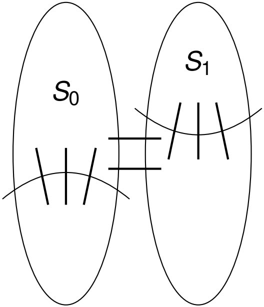
Proof: Induction Step. Case 2.

$$|S_0| \geq |V_0|/2.$$



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Thm: For any cut $(S, V - S)$ in the hypercube, the number of cut edges is at least the size of the small side, $|S|$.
Proof: Induction Step. Case 2.

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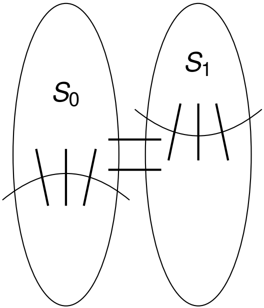

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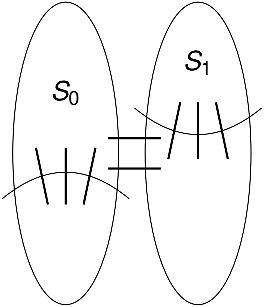

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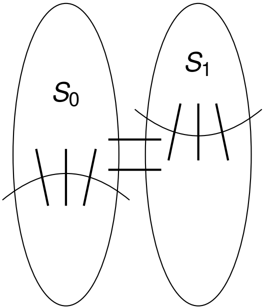

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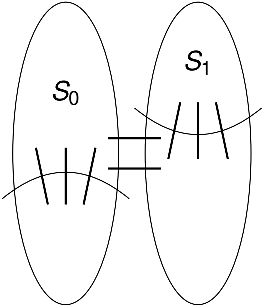

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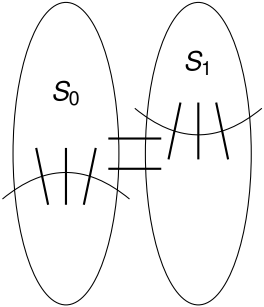

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Edges in E_x connect corresponding nodes.

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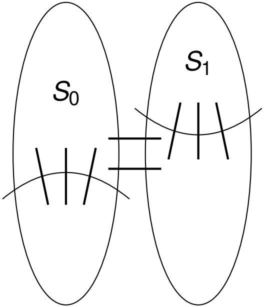

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Edges in E_x connect corresponding nodes.

$\implies = |S_0| - |S_1|$ edges cut in E_x.

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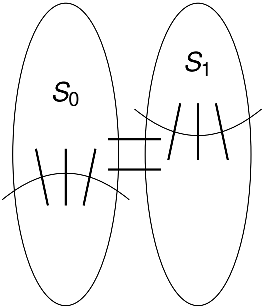

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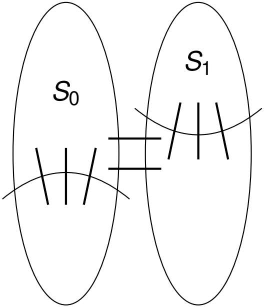

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Edges in E_x connect corresponding nodes.

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Total edges cut:

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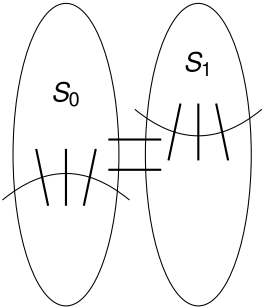

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Total edges cut:
$\geq$

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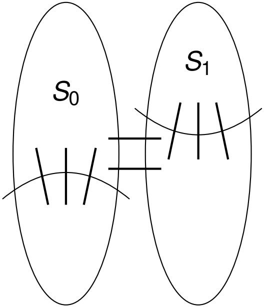

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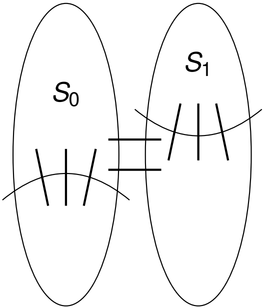

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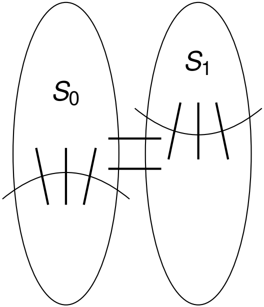

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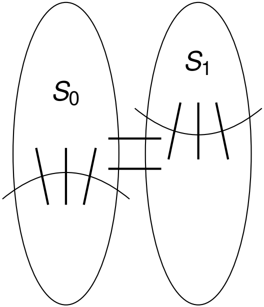

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Total edges cut:

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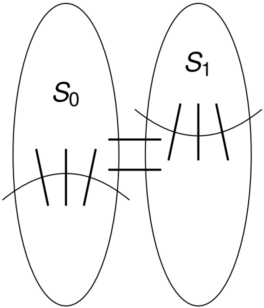

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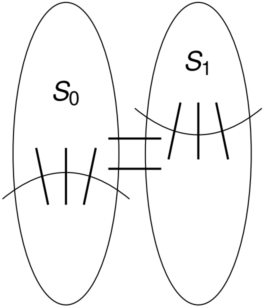

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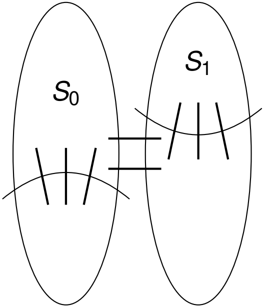

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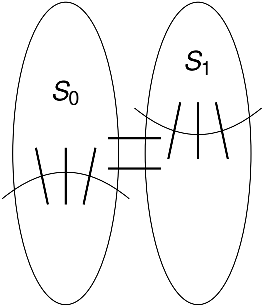

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$|V_0| = |V|/2 \geq |S|$. □

Also, case 3 where $|S_1| \geq |V|/2$ is symmetric.

Hypercubes and Boolean Functions.

The cuts in the hypercubes are exactly the transitions from 0 sets to 1 set on boolean functions on $\{0,1\}^n$.

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Central object of study.

Summary.

We did lots today!

Summary.

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Euler,

Summary.

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Euler, coloring,

Summary.

We did lots today!

Euler, coloring, types of graphs.

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Have a nice weekend!