CS 70 Discrete Mathematics and Probability Theory
Fall 2020

DIS 6B

1 Count it

Let's get some practice with counting!

(a) How many sequences of 15 coin-flips are there that contain exactly 4 heads?

(b) An anagram of HALLOWEEN is any re-ordering of the letters of HALLOWEEN, i.e., any string made up of the letters H, A, L, L, O, W, E, E, N in any order. The anagram does not have to be an English word.
How many different anagrams of HALLOWEEN are there?

(c) How many solutions does $y_0 + y_1 + \cdots + y_k = n$ have, if each y must be a non-negative integer?

(d) How many solutions does $y_0 + y_1 = n$ have, if each y must be a positive integer?

(e) How many solutions does $y_0 + y_1 + \cdots + y_k = n$ have, if each y must be a positive integer?

2 Inclusion and exclusion

What is total number of positive numbers that smaller than 100 and coprime to 100?

3 Identities

(a) $\sum_{i=0}^{n}(-1)^i\binom{n}{i} = 0$

(b) $\sum_{i=0}^{n}\binom{r+i}{i} = \binom{r+n+1}{n}$

(c) $\sum_{i=0}^{n}\binom{r}{i}\binom{s}{n-i} = \binom{r+s}{n}$ (Note: Assuming $r > n, s > n$)

4 Largest binom

For which value(s) of k is $\binom{n}{k}$ maximum? Prove your answer.