

## 1 Continuous Joint Densities

The joint probability density function of two random variables  $X$  and  $Y$  is given by  $f(x,y) = Cxy$  for  $0 \leq x \leq 1, 0 \leq y \leq 2$ , and 0 otherwise (for a constant  $C$ ).

- (a) Find the constant  $C$  that ensures that  $f(x,y)$  is indeed a probability density function.
- (b) Find  $f_X(x)$ , the marginal distribution of  $X$ .
- (c) Find the conditional distribution of  $Y$  given  $X = x$ .
- (d) Are  $X$  and  $Y$  independent?

## 2 Uniform Distribution

You have two fidget spinners, each having a circumference of 10. You mark one point on each spinner as a needle and place each of them at the center of a circle with values in the range  $[0, 10)$  marked on the circumference. If you spin both (independently) and let  $X$  be the position of the first spinner's mark and  $Y$  be the position of the second spinner's mark, what is the probability that  $X \geq 5$ , given that  $Y \geq X$ ?

## 3 Darts with Friends

Michelle and Alex are playing darts. Being the better player, Michelle's aim follows a uniform distribution over a circle of radius  $r$  around the center. Alex's aim follows a uniform distribution over a circle of radius  $2r$  around the center.

- (a) Let the distance of Michelle's throw be denoted by the random variable  $X$  and let the distance of Alex's throw be denoted by the random variable  $Y$ .
- What's the cumulative distribution function of  $X$ ?
  - What's the cumulative distribution function of  $Y$ ?
  - What's the probability density function of  $X$ ?
  - What's the probability density function of  $Y$ ?
- (b) What's the probability that Michelle's throw is closer to the center than Alex's throw? What's the probability that Alex's throw is closer to the center?
- (c) What's the cumulative distribution function of  $U = \min\{X, Y\}$ ?
- (d) What's the cumulative distribution function of  $V = \max\{X, Y\}$ ?
- (e) What is the expectation of the absolute difference between Michelle's and Alex's distances from the center, that is, what is  $\mathbb{E}[|X - Y|]$ ? [*Hint*: Use parts (c) and (d), together with the continuous version of the tail sum formula, which states that  $\mathbb{E}[Z] = \int_0^\infty P(Z \geq z) dz$ .]