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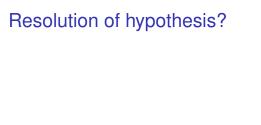
The powerset of a set is the set of all subsets.

Generalized Continuum hypothesis.

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The powerset of a set is the set of all subsets.

Recall: powerset of the naturals is not countable.



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Uh oh....

Naive Set Theory: Any definable collection is a set.

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Axioms changed.

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See Logicomix by Doxiaidis, Papadimitriou (was professor here), Papadatos, Di Donna.

Write me a program checker!

Write me a program checker! Check that the compiler works!

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Check that the compiler works!

How about.. Check that the compiler terminates on a certain input.

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HALT(P, I)

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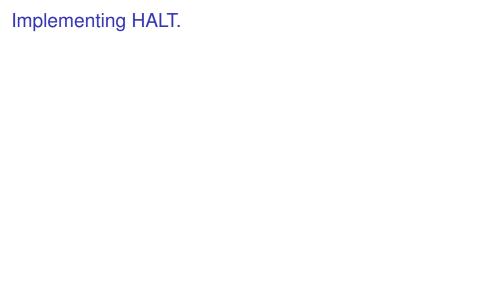
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Run P on I and check!

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Determines if P(I) (P run on I) halts or loops forever.
Run P on I and check!
How long do you wait?
Something about infinity here, maybe?
```



Halt does not exist.

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Proof: Yes!

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What is he talking about? (A) He is confused.

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- (A) He is confused.
- (B) Fermat's Theorem.

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- (A) He is confused.
- (B) Fermat's Theorem.
- (C) Diagonalization.

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- (A) He is confused.
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- (D) Professor is just strange.

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- (A) He is confused.
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- (C). Maybe (D).

Proof:

Proof: Assume there is a program $HALT(\cdot, \cdot)$.

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Turing(P)

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Assumption: there is a program HALT. There is text that "is" the program HALT.

Proof: Assume there is a program $HALT(\cdot, \cdot)$.

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 \implies then HALTS(Turing, Turing) = halts

 \implies Turing(Turing) loops forever.

Turing(Turing) loops forever

Proof: Assume there is a program $HALT(\cdot, \cdot)$.

Turing(P)

- 1. If HALT(P,P) = "halts", then go into an infinite loop.
- 2. Otherwise, halt immediately.

Assumption: there is a program HALT. There is text that "is" the program HALT. There is text that is the program Turing.

Can run Turing on Turing!

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Turing(Turing) loops forever \implies then HALTS(Turing, Turing) \neq halts

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Contradiction. Program HALT does not exist!

Proof: Assume there is a program $HALT(\cdot, \cdot)$.

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Contradiction. Program HALT does not exist! Questions?

Any program is a fixed length string.

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	P_1	P_2	P_3	• • • •
P ₁ P ₂ P ₃	Н	Н	L	
P_2	L	L	Н	
P_3	L	Н	Н	• • •
:	:	:	:	٠

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	P_1	P_2	P_3	• • •
P_1	Н	Н	L	
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P ₁ P ₂ P ₃	L	Н	Н	
:	1	:	:	٠
11.0			•	

Halt - diagonal.

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P_1	Н	Η	L	
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:	:	:	:	٠
'	٠			

Halt - diagonal. Turing - is not Halt.

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	P_1	P_2	P_3	• • •
P.	Н	Н	ı	
P ₁ P ₂ P ₃	Ľ	L	H	
P_3	L	Н	Н	• • •
:	:	:	÷	٠

Halt - diagonal.

Turing - is not Halt. and is different from every P_i on the diagonal.

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P_1	P_2	P_3	• • •
ш	Ц	ı	
L	Ľ	Н	
L	Н	Н	• • •
:	÷	:	٠
	H L L	H H L L L H : :	H H L L L H L H H

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Turing is not on list.

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Turing is not on list. Turing is not a program.

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	P_1	P_2	P_3	• • •
P.	Н	Н	L	
P ₁ P ₂ P ₃	L	L	H	
P_3	L	Н	Н	• • •
÷	:	:	:	٠.

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P_1	Н	Η	L	
P_2	L	L	Н	
P ₁ P ₂ P ₃	L	Н	Н	
:	:	:	:	٠.
•.	٠ ا	٠.	•	•

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÷	:	÷	:	٠

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Halt does not exist!

Assumed HALT(P, I) existed.

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What is P?

Assumed HALT(P, I) existed. What is P? Text.

Assumed HALT(P, I) existed.

What is P? Text.

What is 1?

Assumed HALT(P, I) existed.

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Turing "diagonalizes" on list of program.

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⇒ HALT is not a program.

Questions?

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Turing "diagonalizes" on list of program.
It is not a program!!!!
\implies HALT is not a program.
```

We are so smart!

Wow, that was easy!

We are so smart!

Wow, that was easy!
We should be famous!

In Turing's time.

In Turing's time.

No computers.

In Turing's time.

No computers.

Adding machines.

In Turing's time.

No computers.

Adding machines.

e.g., Babbage (from table of logarithms) 1812.

In Turing's time.

No computers.

Adding machines.

e.g., Babbage (from table of logarithms) 1812.

Concept of program as data wasn't really there.

A Turing machine.

- an (infinite) tape with characters

A Turing machine.

- an (infinite) tape with characters
- be in a state, and read a character

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- an (infinite) tape with characters
- be in a state, and read a character
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Universal Turing machine

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Universal Turing machine

- an interpreter program for a Turing machine

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Now that's a computer!

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Turing: AI,

A Turing machine.

- an (infinite) tape with characters
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Universal Turing machine

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- where the tape could be a description of a ... Turing machine!

Now that's a computer!

Turing: AI, self modifying code,

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- an interpreter program for a Turing machine
- where the tape could be a description of a ... Turing machine!

Now that's a computer!

Turing: AI, self modifying code, learning...

Just a mathematician?

Just a mathematician? "Wrote" a chess program.

Just a mathematician?

"Wrote" a chess program.

Simulated the program by hand to play chess.

Just a mathematician?

"Wrote" a chess program.

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It won!

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It won! Once anyway.

Just a mathematician?

"Wrote" a chess program.

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It won! Once anyway.

Involved with computing labs through the 40s.

Church proved an equivalent theorem. (Previously.)

Church proved an equivalent theorem. (Previously.) Used λ calculus....

Church proved an equivalent theorem. (Previously.) Used λ calculus....which is...

Church proved an equivalent theorem. (Previously.) Used λ calculus....which is... Lisp (Scheme)!!!

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Today:Programs can be written in ascii.

Computing on top of computing...

Computer, assembly code, programming language, browser, html, javascript..

Computing on top of computing...

Computer, assembly code, programming language, browser, html, javascript..

We can't get enough of building more Turing machines.

Does a program, P, print "Hello World"?

Does a program, *P*, print "Hello World"? How?

Does a program, *P*, print "Hello World"? How? What is *P*?

Does a program, *P*, print "Hello World"? How? What is *P*? Text!!!!!!

Does a program, *P*, print "Hello World"? How? What is *P*? Text!!!!!!

Does a program, *P*, print "Hello World"? How? What is *P*? Text!!!!!!

Find exit points and add statement: Print "Hello World."

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Can a set of notched tiles tile the infinite plane?

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Can a set of notched tiles tile the infinite plane? Proof: simulate a computer. Halts if finite.

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Does a set of integer equations have a solution?

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Does a set of integer equations have a solution?

Example: " $x^n + y^n = 1$?"

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Undecidability for Diophantine set of equations \implies no program can take any set of integer equations and

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- Imitation Game.

Tragic ending...

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- British Government apologized (2009) and pardoned (2013).

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Program is text, so we can pass it to itself,
   or refer to self.
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Computer Programs are an interesting thing.

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Computation is a lens for other action in the world.

What's to come?

What's to come? Probability.

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A bag contains:

What's to come? Probability.

A bag contains:

















What's to come? Probability.

A bag contains:



What is the chance that a ball taken from the bag is blue?

What's to come? Probability.

A bag contains:



What is the chance that a ball taken from the bag is blue? Count blue.

What's to come? Probability.

A bag contains:



What is the chance that a ball taken from the bag is blue? Count blue. Count total.

What's to come? Probability.

A bag contains:



What is the chance that a ball taken from the bag is blue? Count blue. Count total. Divide.

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Next Up: Probability.