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Disjoint – so add!

CS70: On to probability.

Modeling Uncertainty: Probability Space

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Modeling Uncertainty: Probability Space

1. Key Points
2. Random Experiments
3. Probability Space

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  - Optimizes use of knowledge to make decisions

The Magic of Probability

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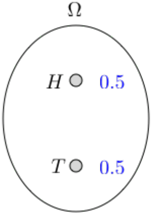
Physical Experiment



Probability Model

Random Experiment: Flip one Fair Coin

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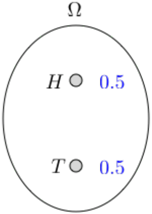

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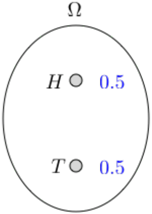

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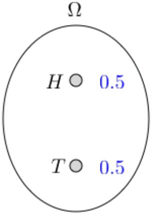

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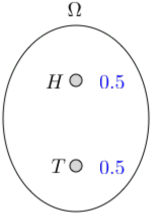

Probability Model

- The physical experiment is complex. (Shape, density, initial momentum and position, ...)
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Random Experiment: Flip one Fair Coin

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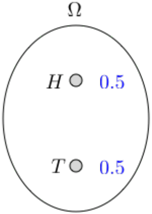

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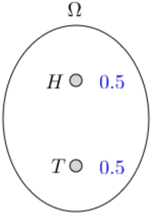

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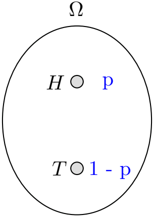

Probability Model

Flip Two Fair Coins

Flip Two Fair Coins

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Flip Two Fair Coins

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Flip Two Fair Coins

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Flips two coins glued together side by side:

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Glued coins

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- Note: Coins are glued so that they show different faces.

Flip two Attached Coins

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Flips two coins attached by a spring:

Flip two Attached Coins

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- Note: Coins are attached so that they tend to show the same face, unless the spring twists enough.

Flipping Two Coins

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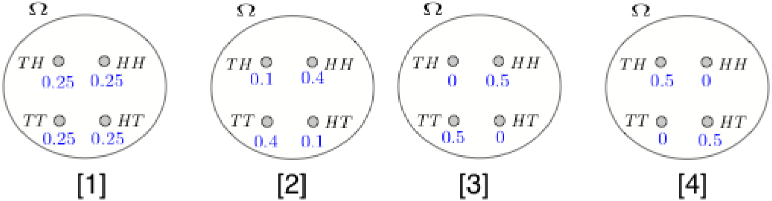

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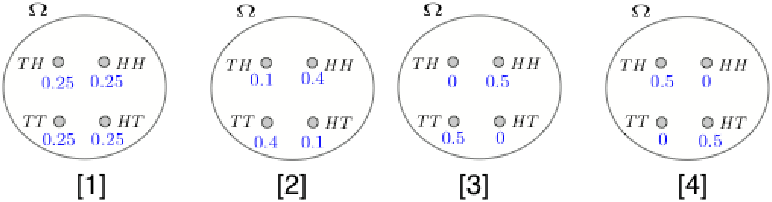

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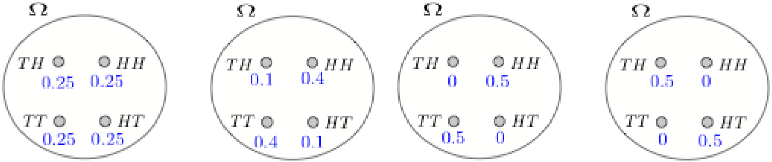

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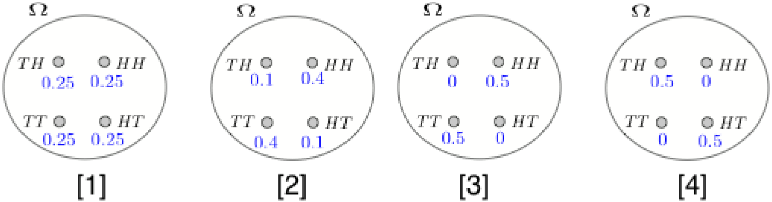

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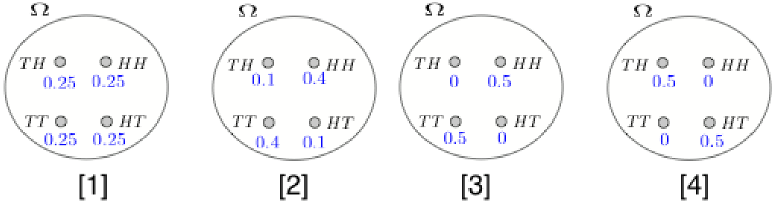

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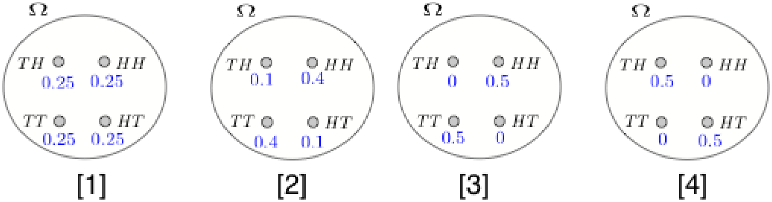

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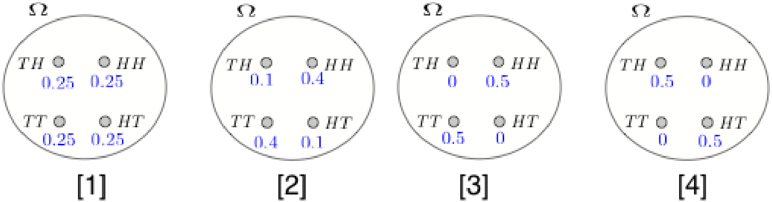

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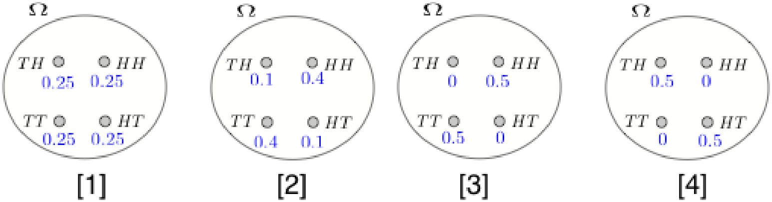

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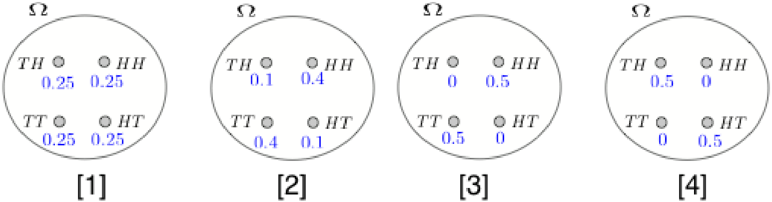

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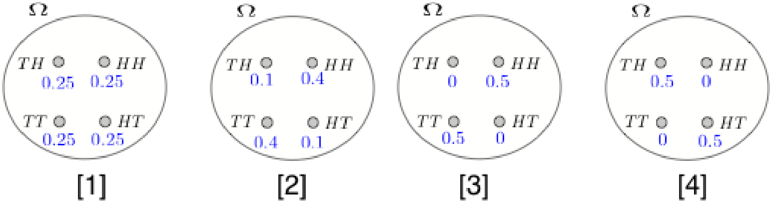

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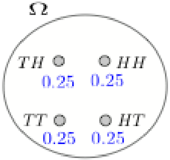

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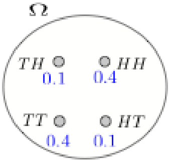

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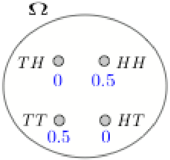

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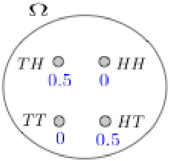

[4]

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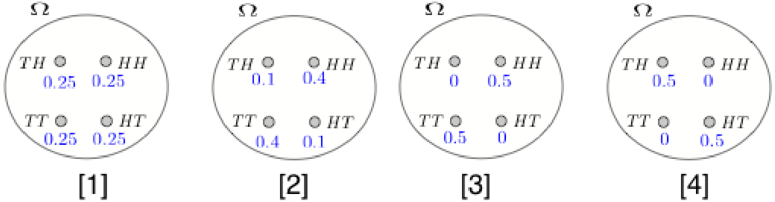

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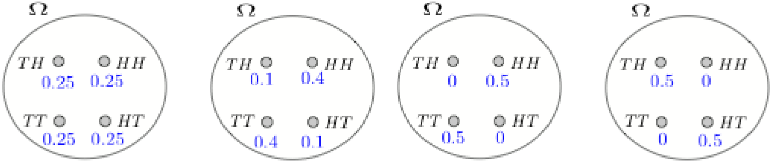

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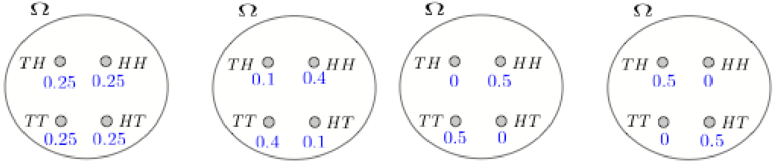

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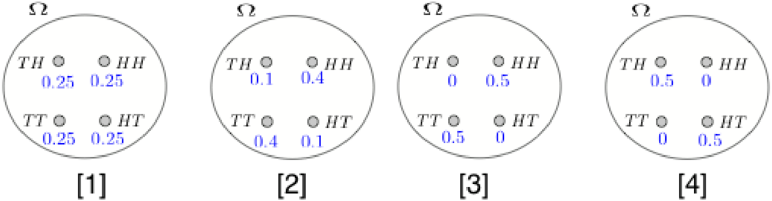

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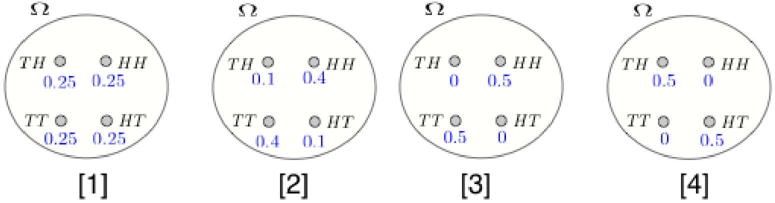

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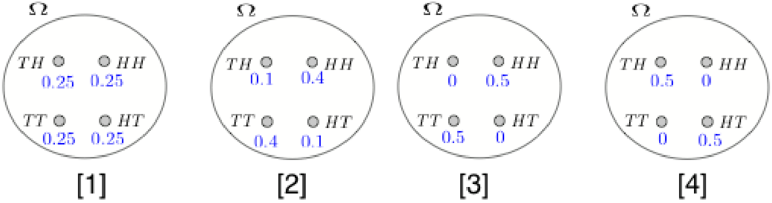

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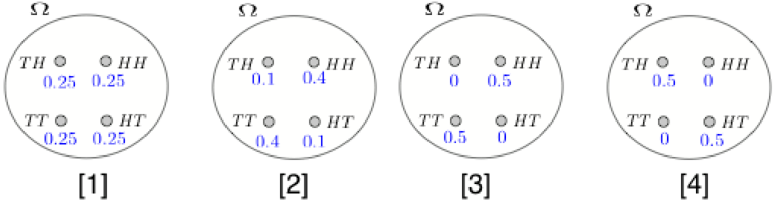

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- Ω and the probabilities specify the random experiment.

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Flip a fair coin n times (some $n \geq 1$):

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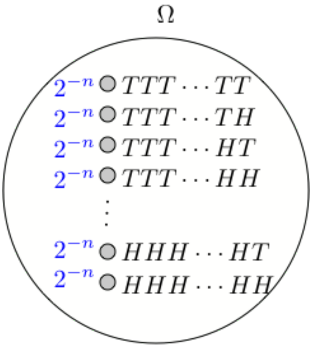

Roll two Dice

Roll a balanced 6-sided die twice:

Roll two Dice

Roll a balanced 6-sided die twice:

- Possible outcomes:

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Roll a balanced 6-sided die twice:

- Possible outcomes: $\{1, 2, 3, 4, 5, 6\}^2 = \{(a, b) \mid 1 \leq a, b \leq 6\}$.

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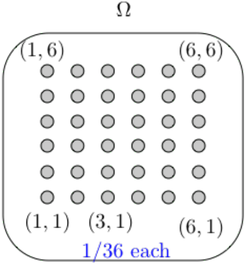

Probability Model

Probability Space.

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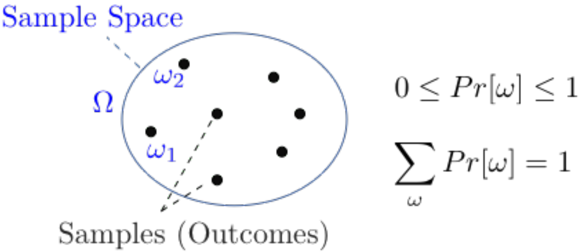

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In a **uniform probability space** each outcome ω is equally probable: $Pr[\omega] = \frac{1}{|\Omega|}$ for all $\omega \in \Omega$.

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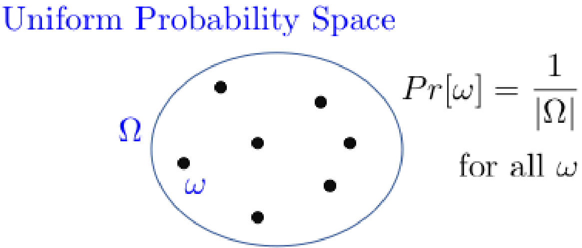

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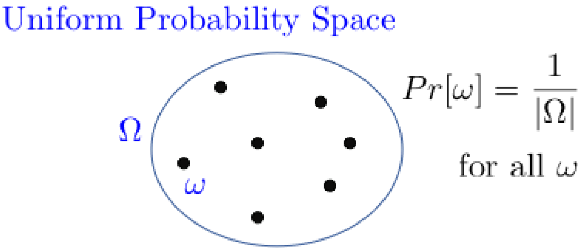

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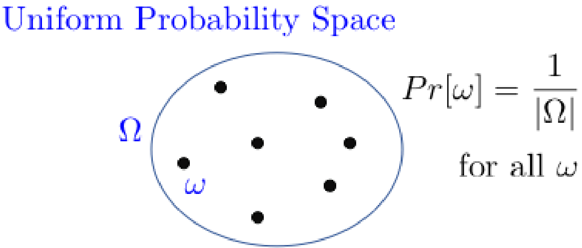

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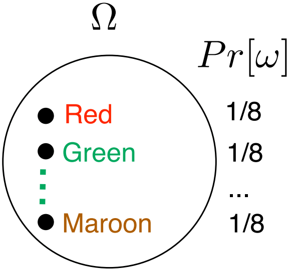
Simplest physical model of a uniform probability space:

Probability Space: Formalism

Simplest physical model of a uniform probability space:

Physical experiment



Probability model

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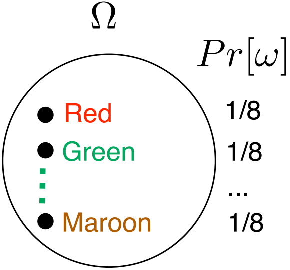

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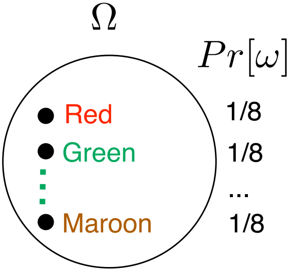

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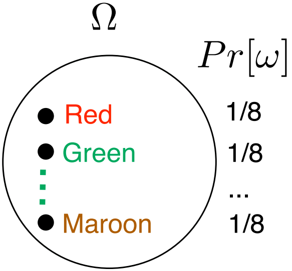

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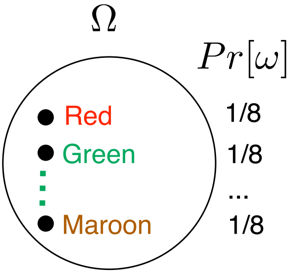

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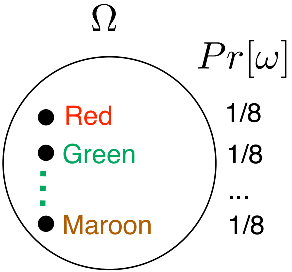

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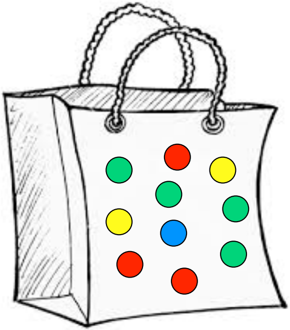

Physical experiment

Ω

ω	$Pr[\omega]$
Red	3/10
Green	4/10
Yellow	2/10
Blue	1/10

Probability model

Probability Space: Formalism

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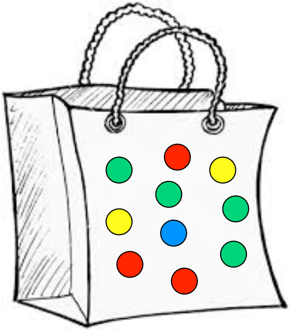

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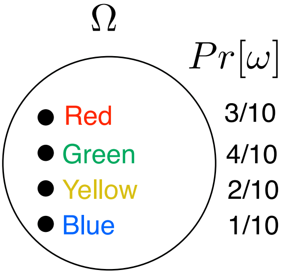

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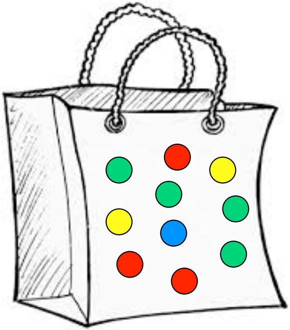

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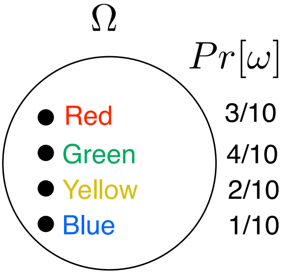

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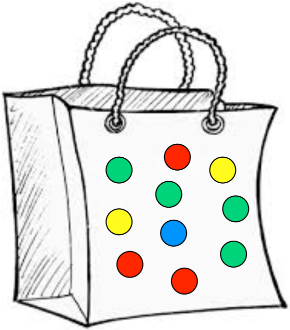

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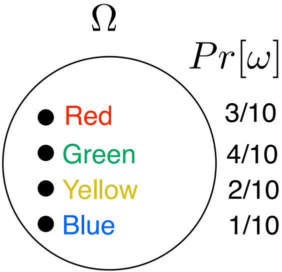

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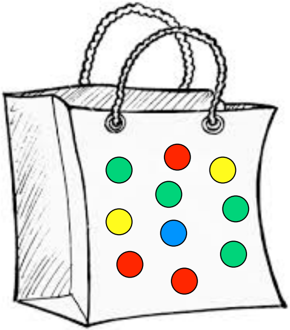

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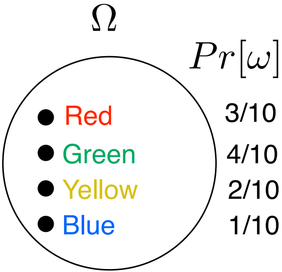

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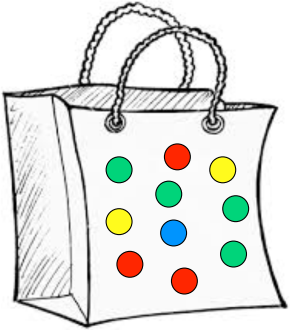

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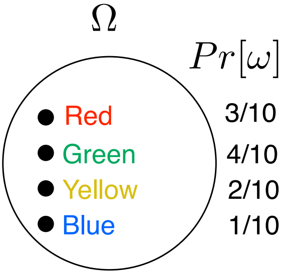

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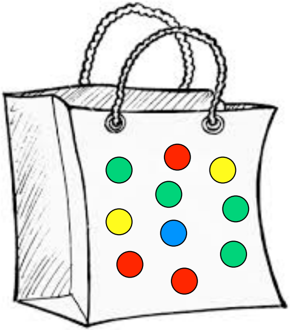

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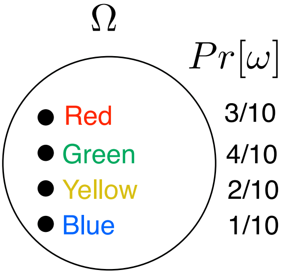

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Note: Probabilities are restricted to rational numbers: $\frac{N_k}{N}$.

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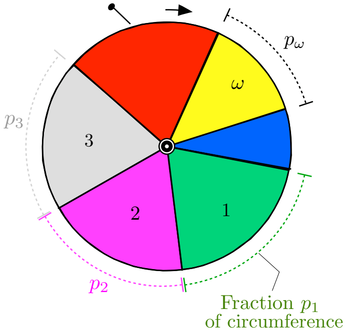

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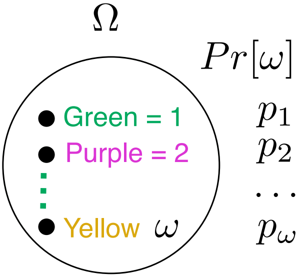

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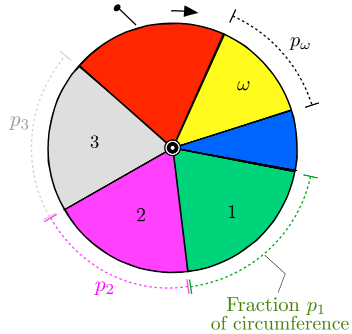

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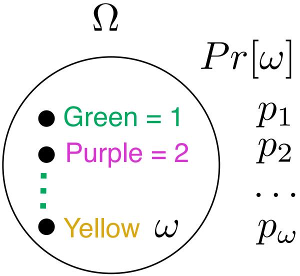

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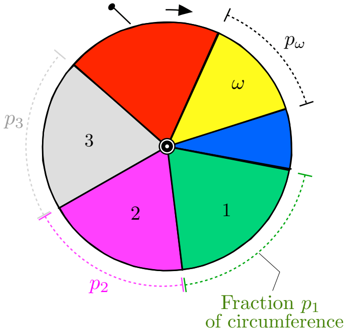

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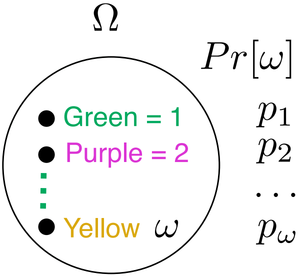

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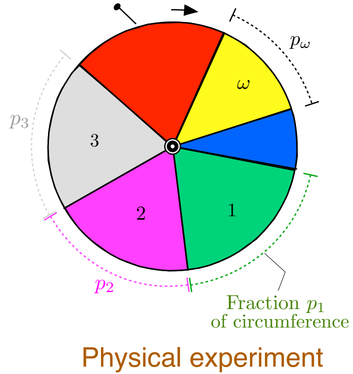

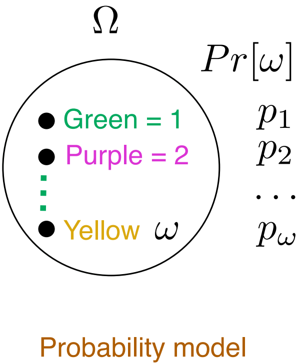

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Summary of Probability Basics

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Onwards in Probability.

Events, Conditional Probability, Independence, Bayes' Rule

CS70: On to Events.

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Events, Conditional Probability, Independence, Bayes' Rule

Today: Events.

Probability Basics Review

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Probability Basics Review

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Set notation review

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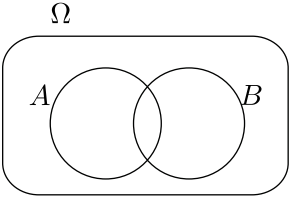
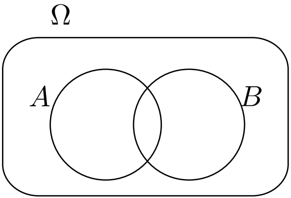


Figure: Two events

Set notation review



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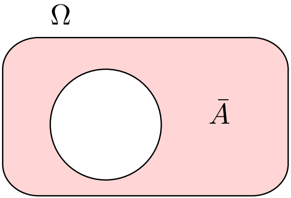
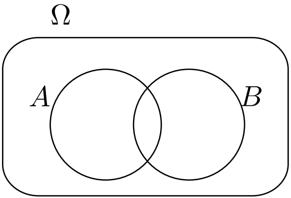


Figure: Complement (not)

Set notation review



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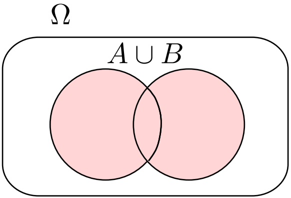
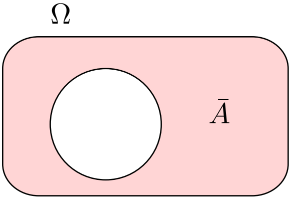


Figure: Union (or)



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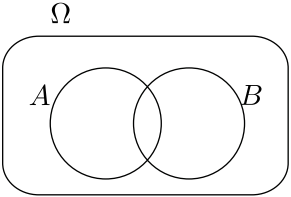

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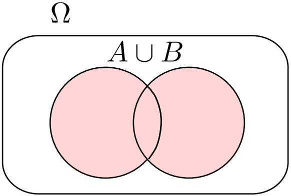

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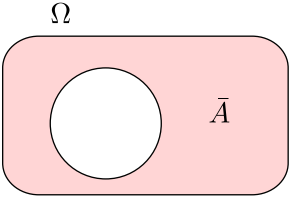

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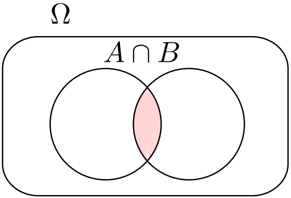

Figure: Intersection (and)

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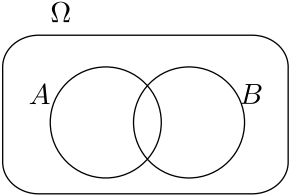

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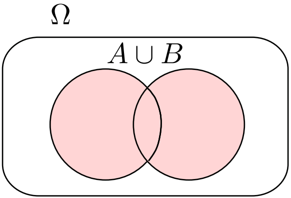

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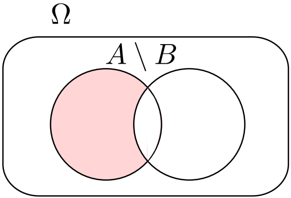
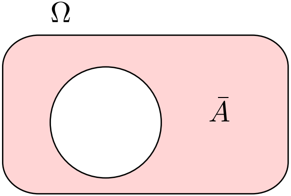


Figure: Difference (*A*, not *B*)



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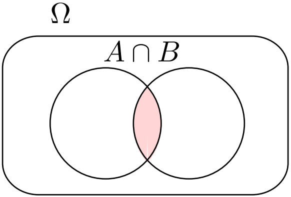

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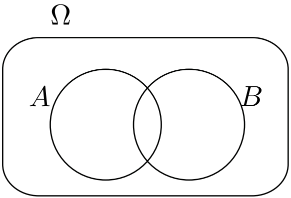

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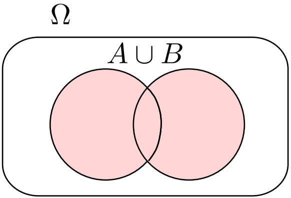

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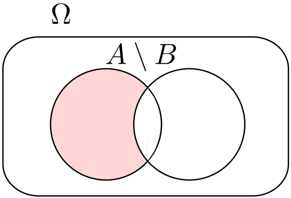

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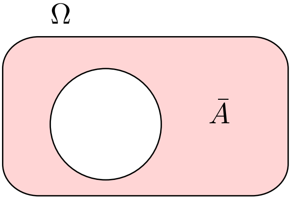

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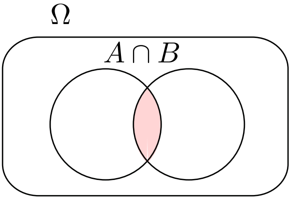

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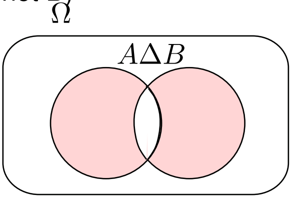

Figure: Symmetric difference (only one)

Probability of exactly one ‘heads’ in two coin flips?

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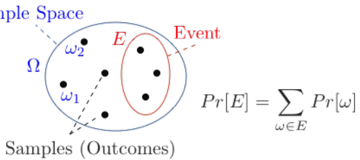

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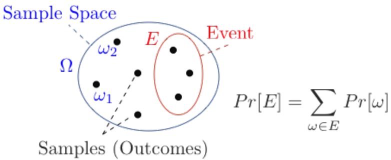

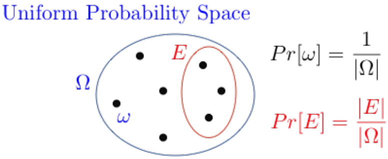

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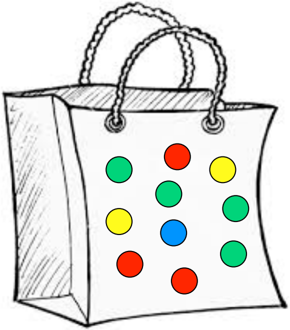
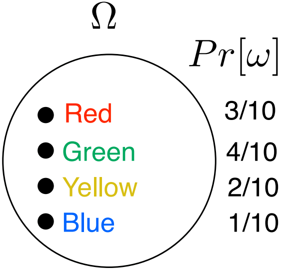


Physical experiment



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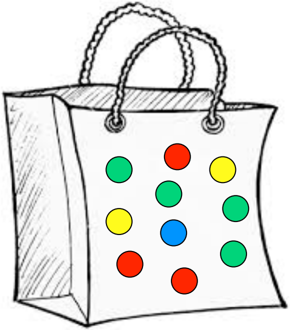

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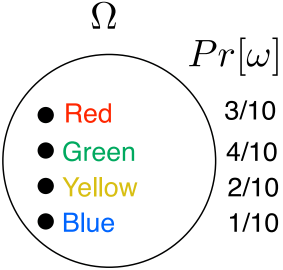

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$\Omega = \{$Red, Green, Yellow, Blue$\}$

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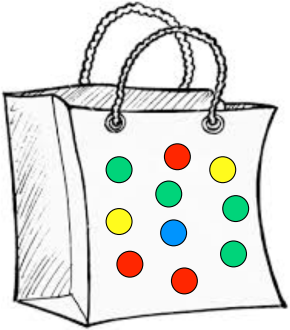

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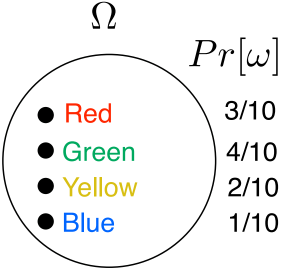

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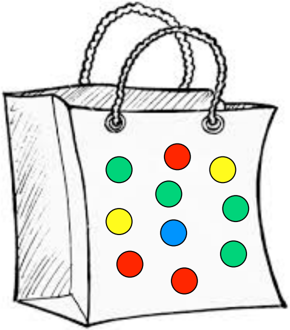

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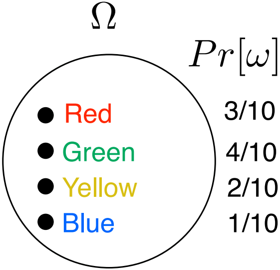

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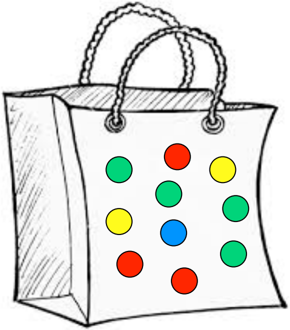

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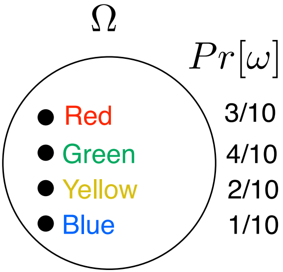

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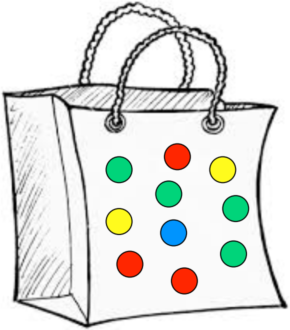

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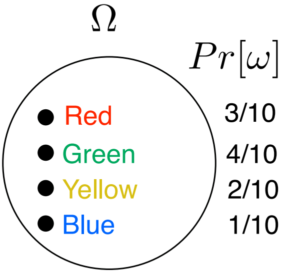

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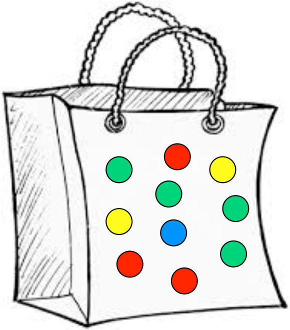

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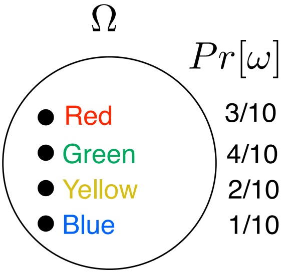

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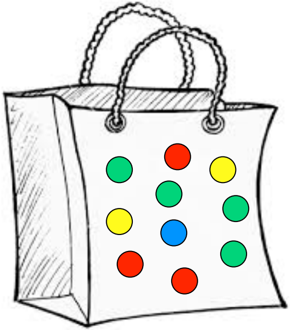

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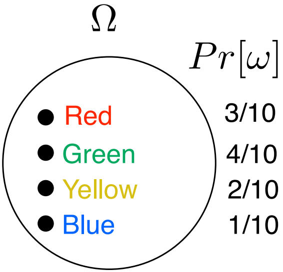

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$E = \{Red, Green\} \Rightarrow Pr[E] =$

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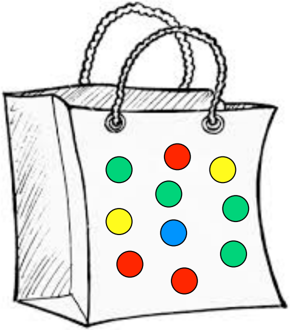

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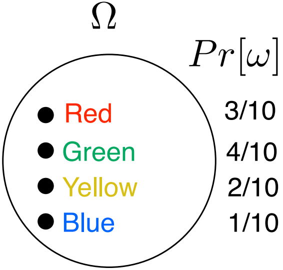

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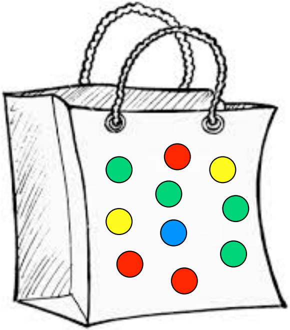

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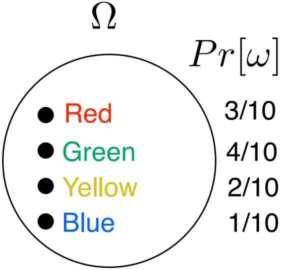

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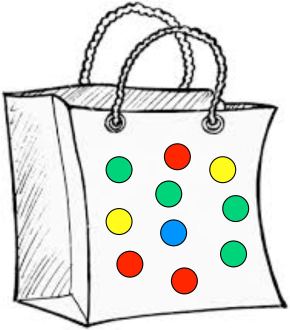

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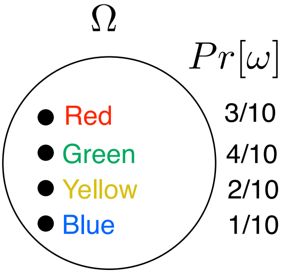

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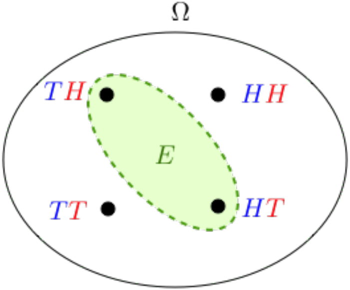

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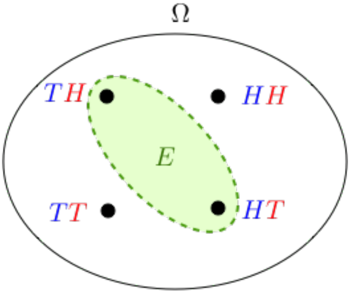

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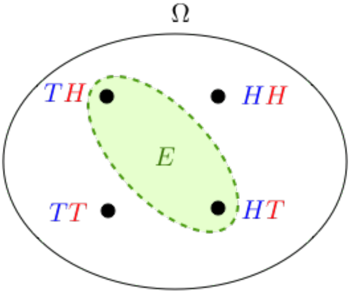

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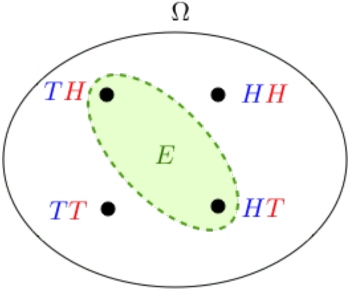

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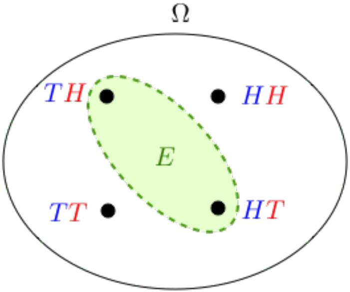
Probability of exactly one heads in two coin flips?

Sample Space, $\Omega = \{HH, HT, TH, TT\}$.

Uniform probability space: $Pr[HH] = Pr[HT] = Pr[TH] = Pr[TT] = \frac{1}{4}$.

Event, E, "exactly one heads": $\{TH, HT\}$.



$$Pr[E] = \sum_{\omega \in E} Pr[\omega] = \frac{|E|}{|\Omega|} = \frac{2}{4} = \frac{1}{2}.$$

Roll a red and a blue die.

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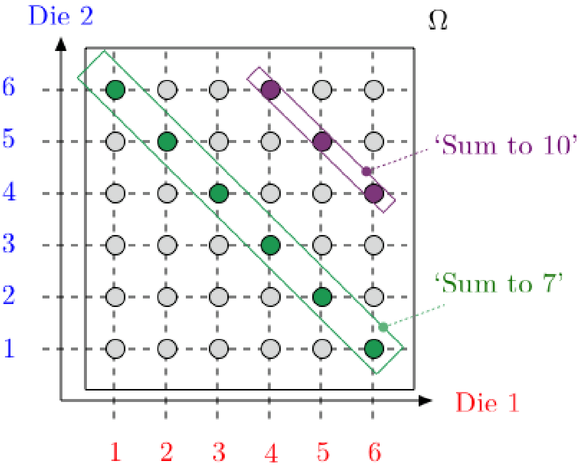

$Pr[\text{Sum to 7}] = \frac{6}{36}$ $Pr[\text{Sum to 10}] = \frac{3}{36}$

Example: 20 coin tosses.

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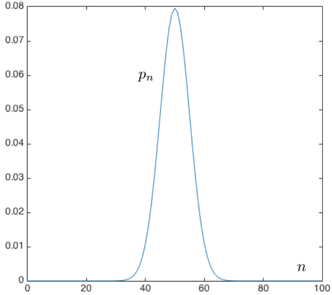

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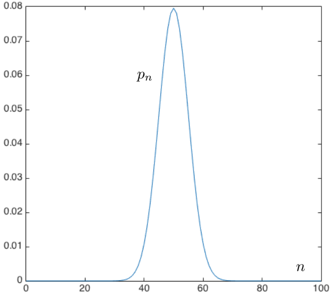

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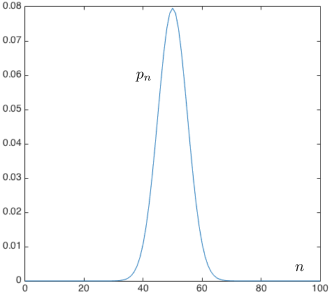

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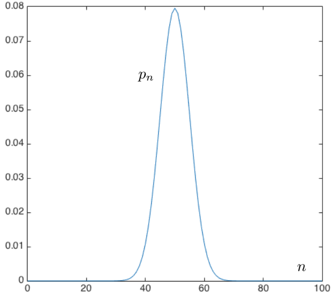

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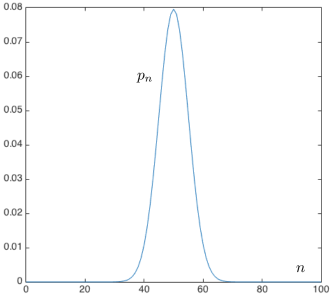

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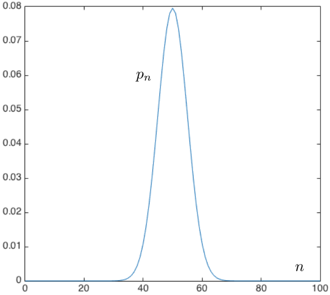

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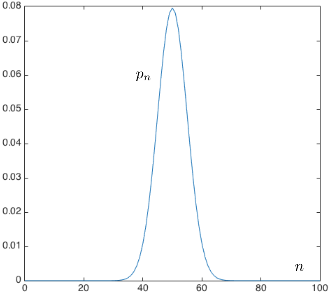

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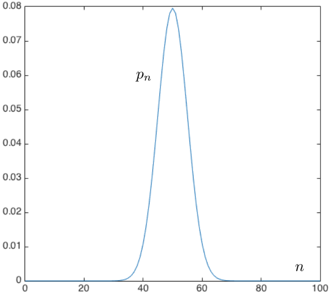

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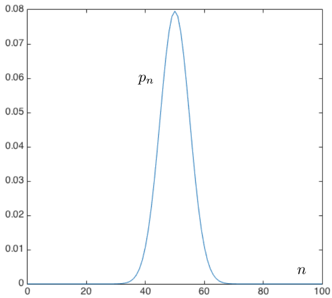

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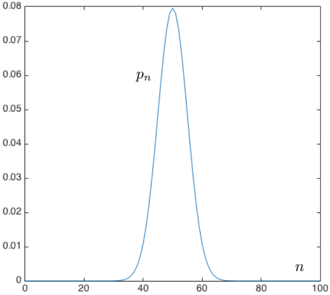

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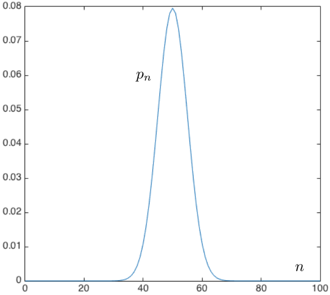

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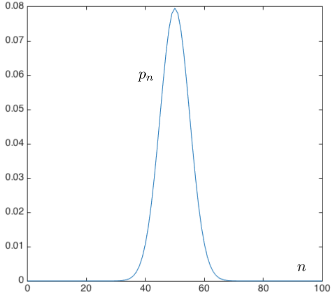

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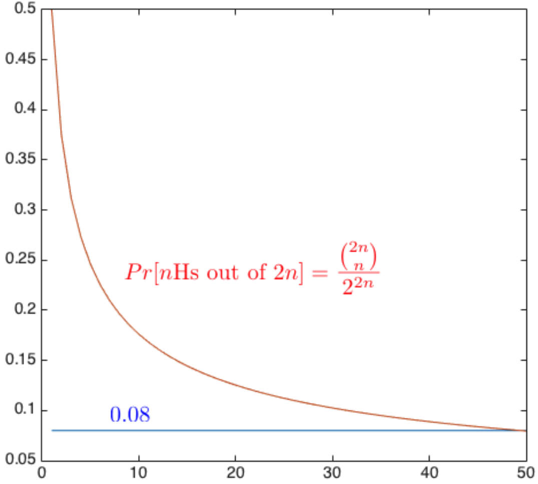
Stirling formula (for large n):

$$n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n.$$

$$\binom{2n}{n} \approx \frac{\sqrt{4\pi n}(2n/e)^{2n}}{[\sqrt{2\pi n}(n/e)^n]^2} \approx \frac{4^n}{\sqrt{\pi n}}.$$

$$Pr[E] = \frac{|E|}{|\Omega|} = \frac{|E|}{2^{2n}} = \frac{1}{\sqrt{\pi n}} = \frac{1}{\sqrt{50\pi}} \approx .08.$$

Exactly 50 heads in 100 coin tosses.



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4. Event: "subset of outcomes." $A \subseteq \Omega$. $Pr[A] = \sum_{w \in A} Pr[\omega]$
5. Some calculations.