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Drink $\implies$ "≥ 18"

"< 18" $\implies$ Don't Drink.

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$\wedge$,$\vee$, $\neg$, $P \implies Q \equiv \neg P \vee Q$.

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Truth Table. Putting together identities. (E.g., cases, substitution.)

CS70: Lecture 2. Outline.

Today: Proofs!!!

1. By Example.
2. Direct. (Prove $P \implies Q$.)
3. by Contraposition (Prove $P \implies Q$)
4. by Contradiction (Prove P.)
5. by Cases

If time: discuss induction.

Quick Background and Notation.

Integers closed under addition.

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$a, b \in Z \implies a + b \in Z$

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A natural number $p > 1$, is **prime** if it is divisible only by 1 and itself.

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

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$b = aq$

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$b-c = aq - aq'$

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Goal: $P \implies Q$
Assume P.
$\ldots$
Therefore Q.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

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$n = 121$ Alt Sum: $1 - 2 + 1 = 0$.

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Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

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Add $99a + 11b$ to both sides.

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$$100a + 10b + c = 11k + 99a + 11b$$

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Left hand side is n,

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Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

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Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

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- Proof assumed no primes *in between* p_k and q.

Proof by cases.

Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.

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Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

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Case 1: a odd, b odd: odd - odd +odd = even.

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Multiply by b^5,

$$a^5 - ab^4 + b^5 = 0$$

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Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.
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Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

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The fourth case is the only one possible, so the lemma follows. $\square$

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

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$$x^y =$$

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Question: Which case holds?

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Question: Which case holds? Don't know!!!

Be careful.

Theorem: $3 = 4$

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Proof: Assume $3 = 4$.

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Don't assume what you want to prove!

Be really careful!

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Proof:

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\begin{aligned}
(x^2 - xy) &= x^2 - y^2 \\
x(x - y) &= (x + y)(x - y)
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$P \implies Q$ does not mean $Q \implies P$.

Summary: Note 2.

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CS70: Note 3. Induction!

Poll.

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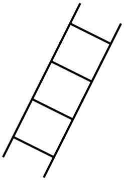
Poll.

1. The natural numbers.
2. 5 year old Gauss.
3. ..and Induction.
4. Simple Proof.

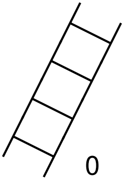
The natural numbers.

The natural numbers.



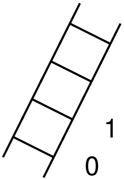
The natural numbers.

0,


0

The natural numbers.

0, 1,



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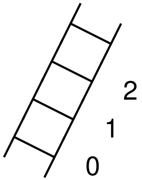

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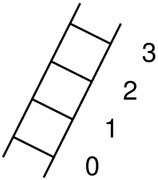

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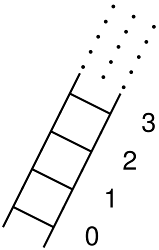

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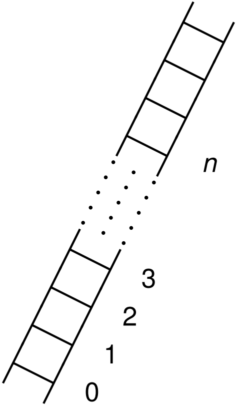

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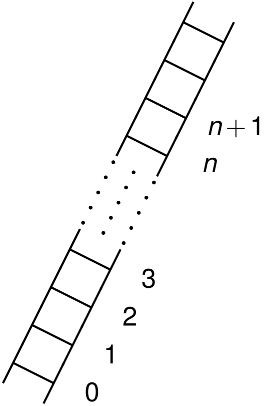

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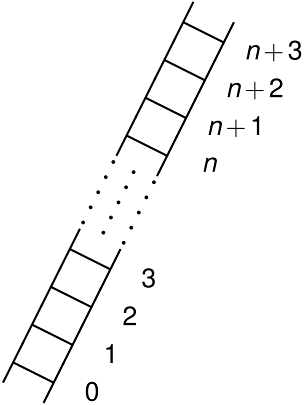

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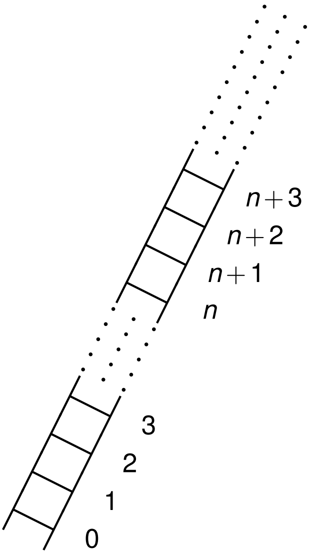

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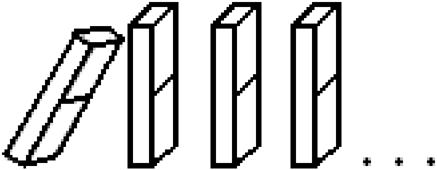
Notes visualization

Note's visualization: an infinite sequence of dominos.



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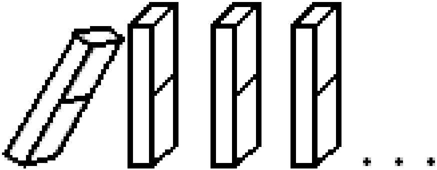

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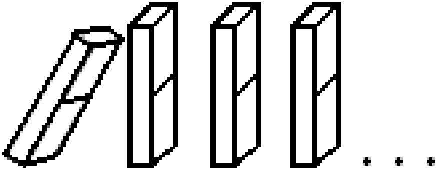

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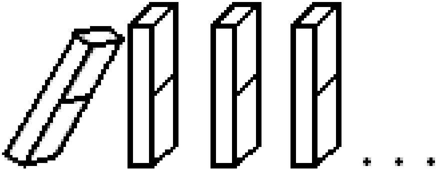

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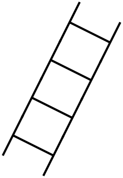

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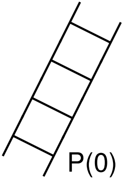

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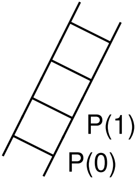

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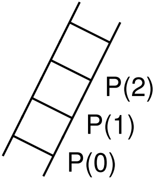

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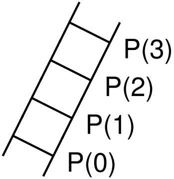

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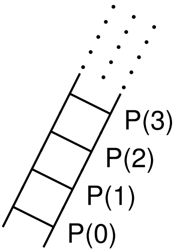

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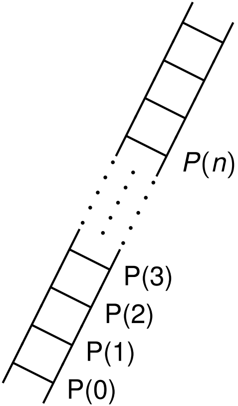

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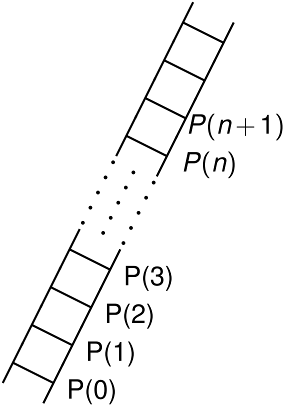

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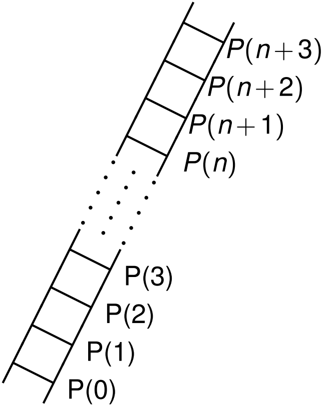

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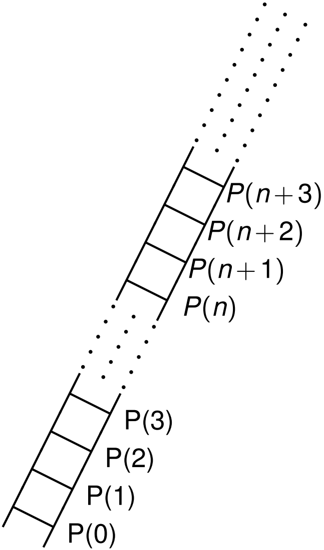

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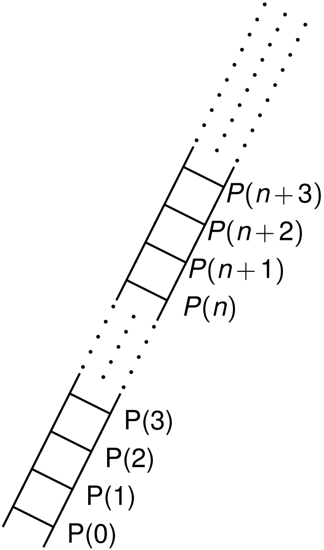

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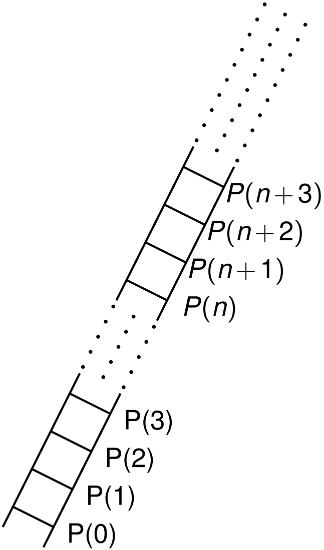

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Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$

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Gauss and Induction

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Predicate, $P(n)$, True for all natural numbers! **Proof by Induction.**

Induction

The canonical way of proving statements of the form

$$(\forall k \in N)(P(k))$$

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- Prove $P(0)$. “Base Case”.

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The basic form

- Prove $P(0)$. "Base Case".
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The canonical way of proving statements of the form

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- For all natural numbers n, $1+2\cdots n = \frac{n(n+1)}{2}$.
- For all $n \in N$, $n^3 - n$ is divisible by 3.
- The sum of the first n odd integers is a perfect square.

The basic form

- Prove $P(0)$. "Base Case".
- $P(k) \implies P(k+1)$
  - Assume $P(k)$, "Induction Hypothesis"

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Next Time.

More induction!

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See you on Thursday!