Today.

Principle of Induction.(continued.)

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And we get...

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Climb an infinite ladder?

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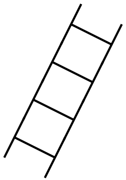
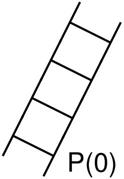


Climb an infinite ladder?

$P(0)$



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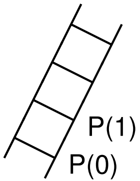

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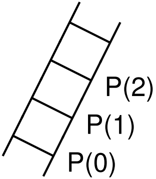

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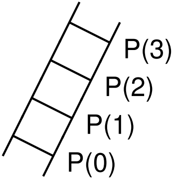

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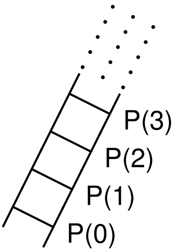

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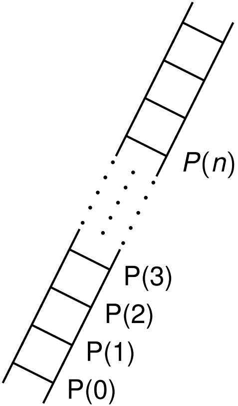

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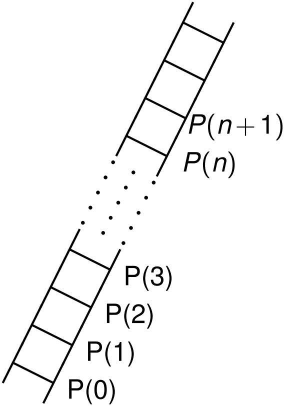

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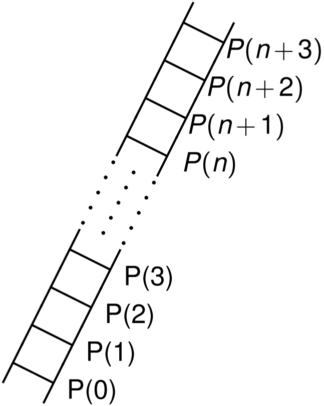

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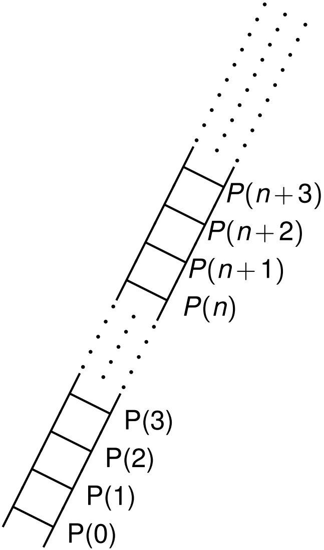

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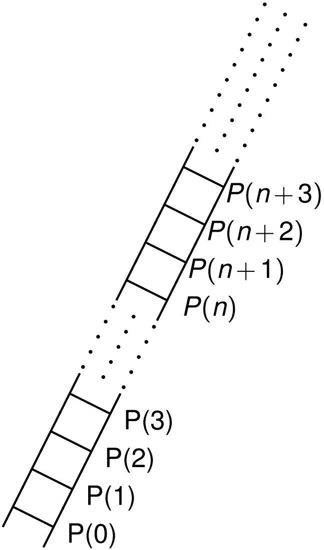

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Your favorite example of forever..

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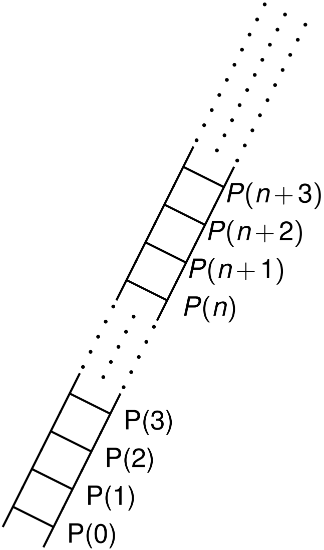

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Your favorite example of forever..or the natural numbers...

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=0}^{n} i = \frac{n(n+1)}{2})$

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How about $k+2$.

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Is this a proof? It shows that we can always move to the next step.

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Gauss and Induction

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Gauss and Induction

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plus inductive step

Gauss and Induction

Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=0}^{n} i = \frac{n(n+1)}{2})$ Proof?

Idea: assume predicate $P(n)$ for $n = k$. $P(k)$ is $\sum_{i=0}^{k} i = \frac{k(k+1)}{2}$.

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Statement is true for $n = 0$ $P(0)$ is true

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Predicate, $P(n)$, True for all natural numbers! **Proof by Induction.**

Another Induction Proof.

Theorem: For every $n \in N$, $n^3 - n$ is divisible by 3. ($3|(n^3 - n)$).

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$(k+1)^3 - (k+1)$

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$(q + k^2 + k)$ is integer (closed under addition and multiplication).

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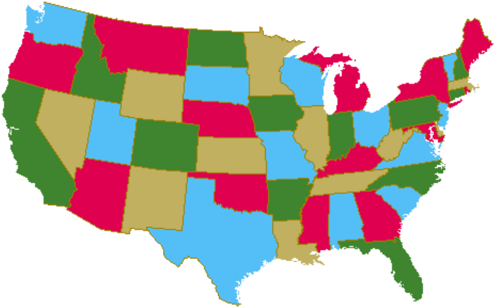
Four Color Theorem.

Theorem: Any map can be colored so that those regions that share an edge have different colors.



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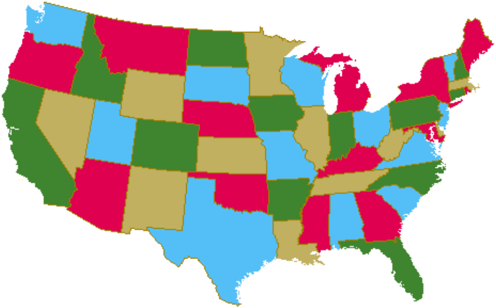

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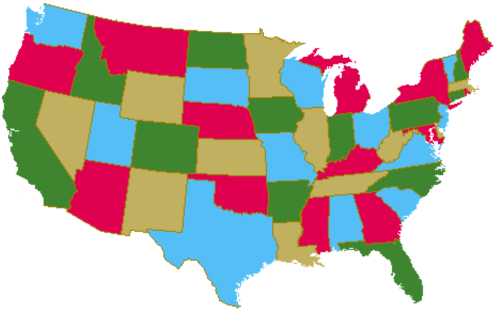

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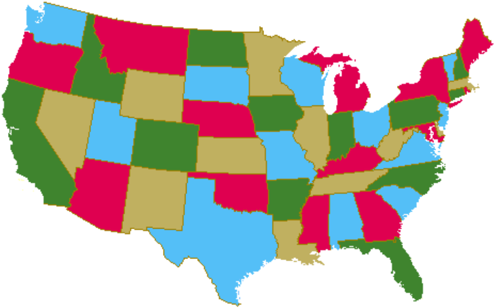

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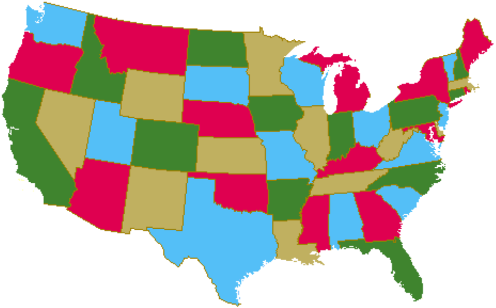

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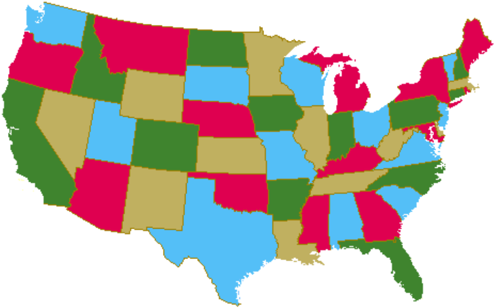

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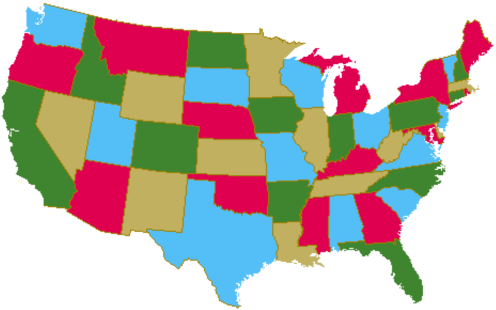

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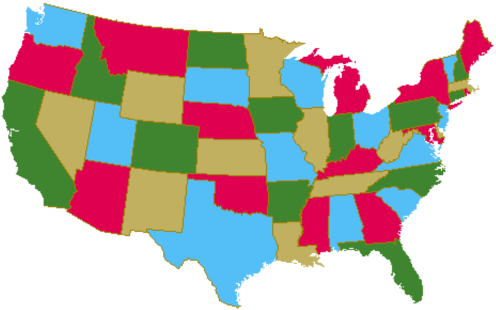

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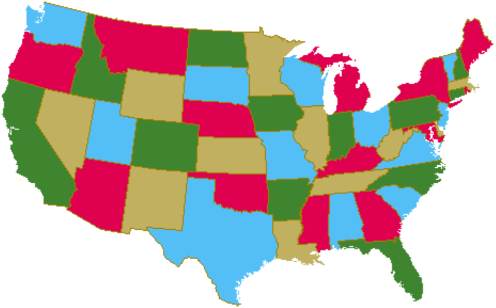

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Two color theorem: example.

Any map formed by dividing the plane into regions by drawing straight lines can be properly colored with two colors.

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Proper coloring: for each line segment the regions on the two sides have different colors.1

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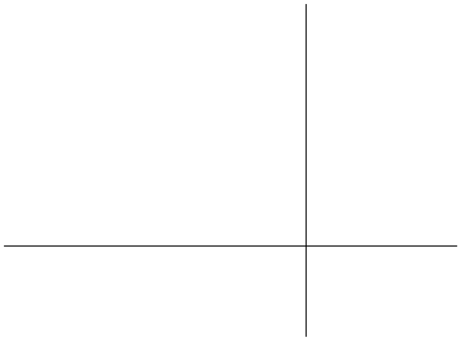

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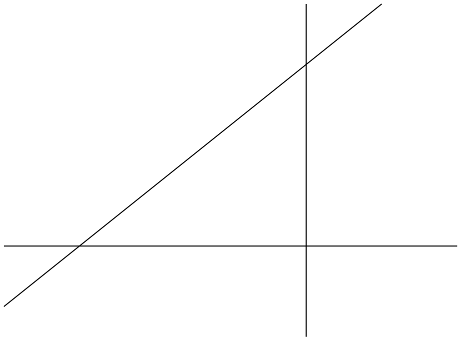

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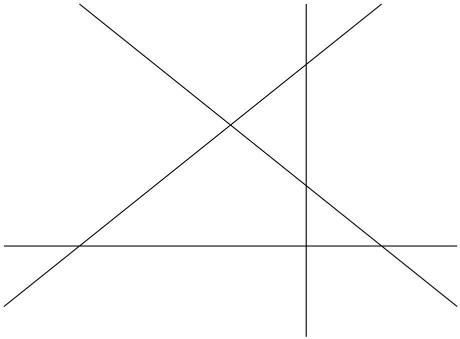

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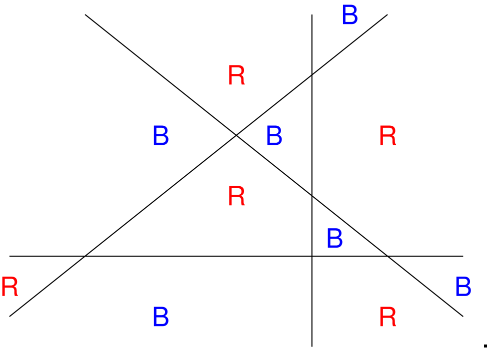

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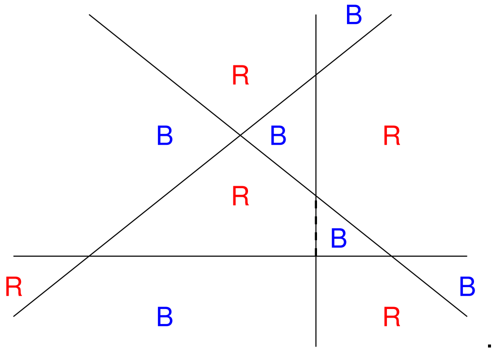

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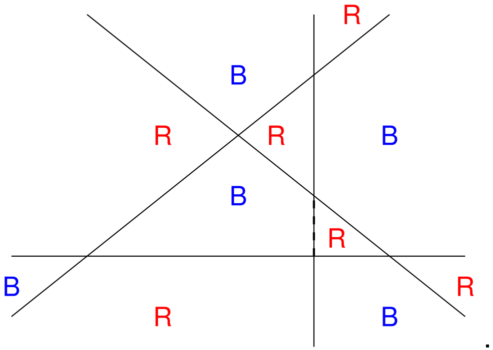

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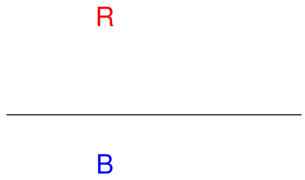

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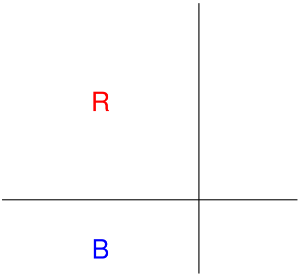

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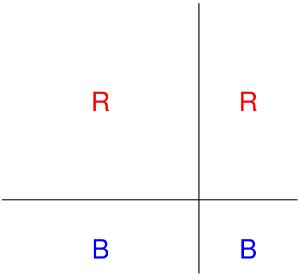

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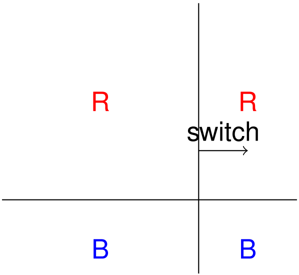

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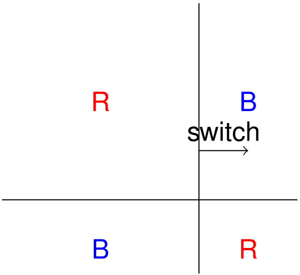

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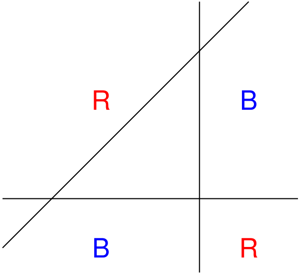

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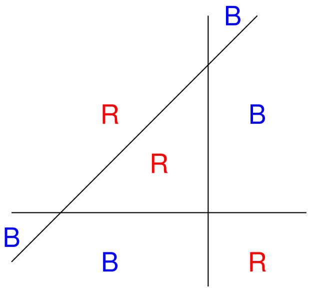

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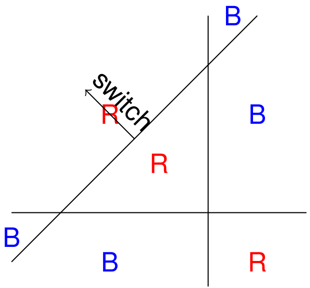

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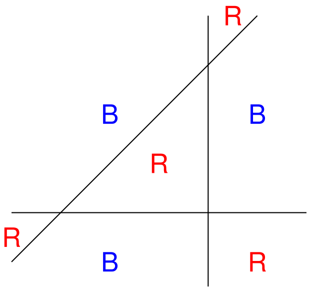

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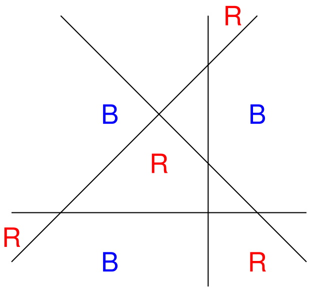

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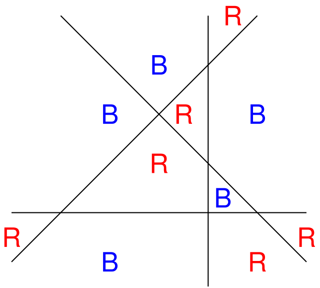

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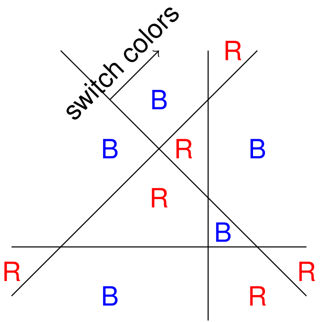

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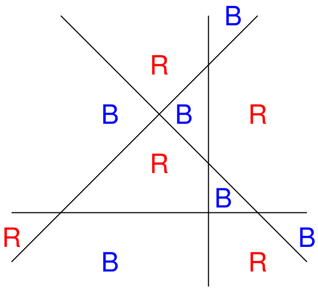

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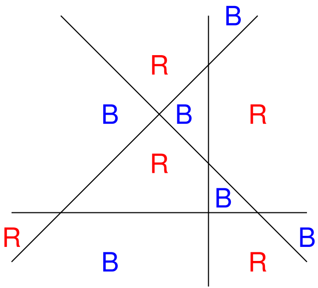

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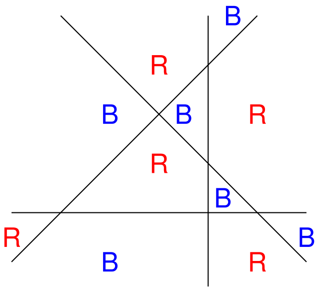

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Theorem: The sum of the first n odd numbers is a perfect square.

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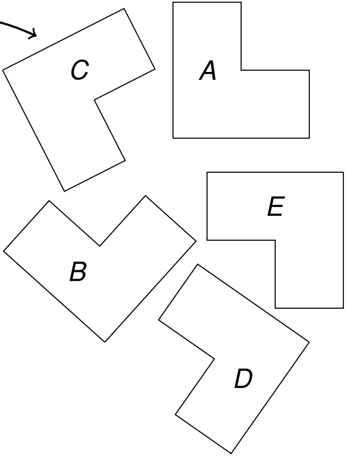
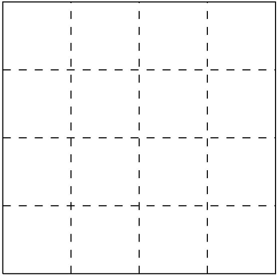
$\square$

Tiling Cory Hall Courtyard.

Use these L-tiles.

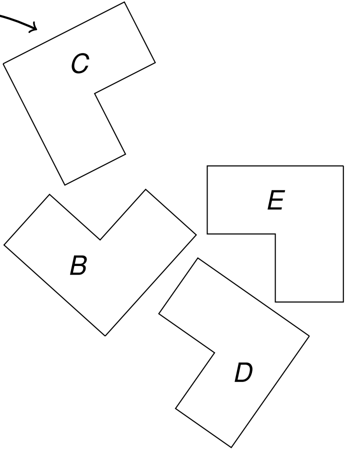


To Tile this 4×4 courtyard.



Tiling Cory Hall Courtyard.

Use these L-tiles.



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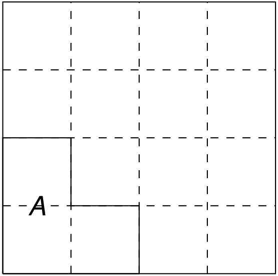

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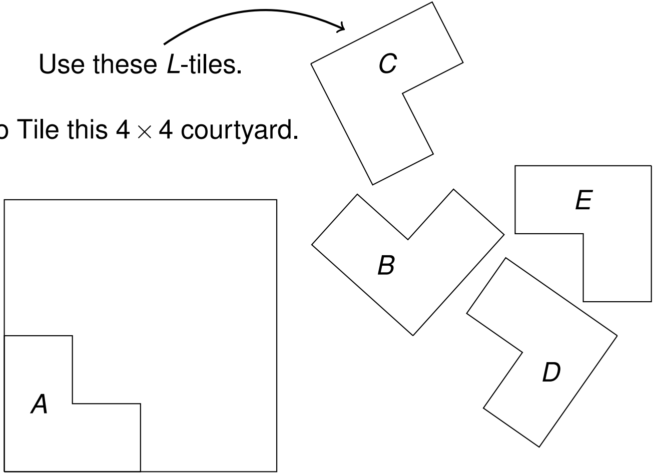

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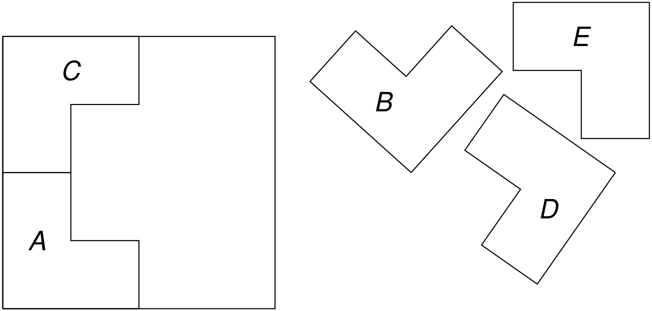

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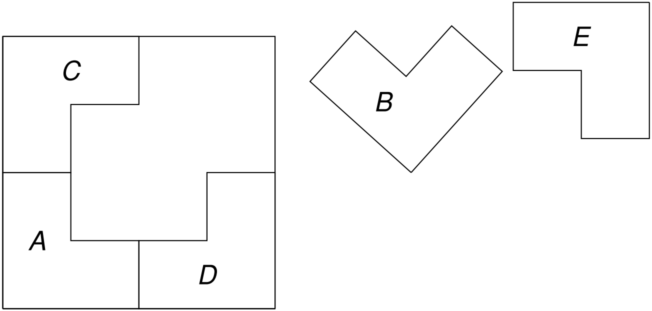

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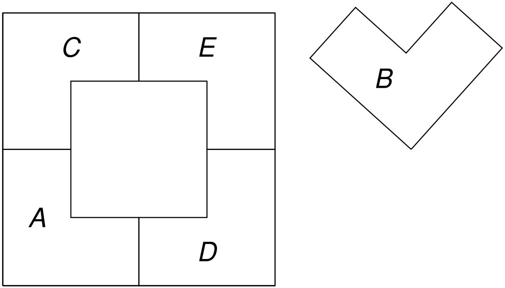

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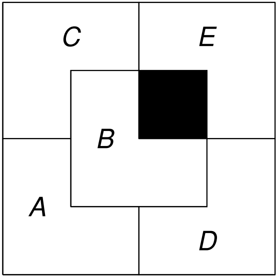

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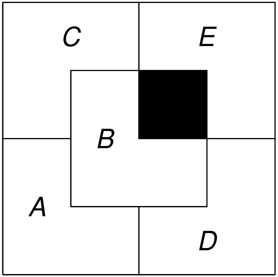

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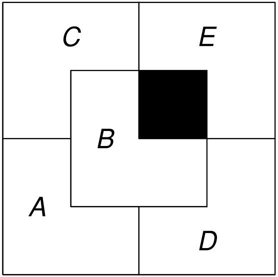


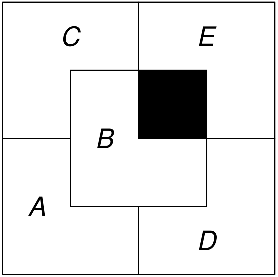
Alright!

Tiled 4×4 square with 2×2 L-tiles.

Tiling Cory Hall Courtyard.

Use these L-tiles.

To Tile this 4×4 courtyard.



Alright!

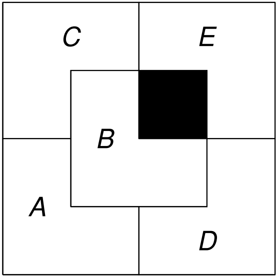
Tiled 4×4 square with 2×2 L-tiles.

with a center hole.

Tiling Cory Hall Courtyard.

Use these L-tiles.

To Tile this 4×4 courtyard.



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Can we tile any $2^n \times 2^n$ with L-tiles (with a hole)

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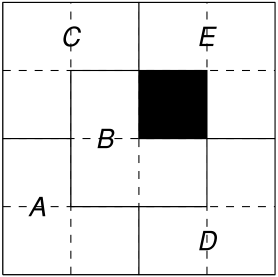

Alright!

Tiled 4×4 square with 2×2 L-tiles.

with a center hole.

Can we tile any $2^n \times 2^n$ with L-tiles (with a hole) for every n!

Hole have to be there? Maybe just one?

Theorem: Any tiling of $2^n \times 2^n$ square has to have one hole.

Hole have to be there? Maybe just one?

Theorem: Any tiling of $2^n \times 2^n$ square has to have one hole.

Proof: The remainder of 2^{2n} divided by 3 is 1.

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$$2^{2(k+1)}$$

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$$2^{2(k+1)} = 2^{2k} * 2^2$$

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Hole in center?

Theorem: Can tile the $2^n \times 2^n$ square to leave a hole adjacent to the center.
Proof:

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The hole is adjacent to the center of the 2×2 square.

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Any $2^n \times 2^n$ square can be tiled with a hole at the center.

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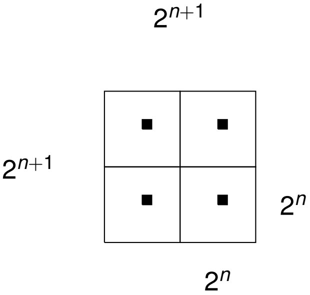

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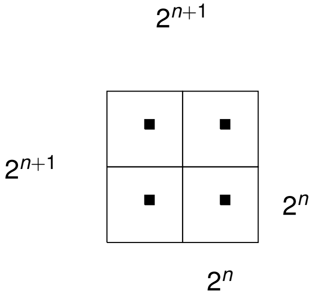

What to do now???

Hole can be anywhere!

Theorem: Can tile the $2^n \times 2^n$ to leave a hole adjacent *anywhere.*

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Better theorem

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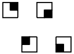

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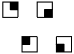

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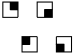

Induction Hypothesis:
"Any $2^n \times 2^n$ square can be tiled with a hole **anywhere.**"
Consider $2^{n+1} \times 2^{n+1}$ square.

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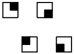

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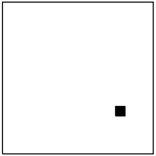

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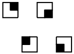

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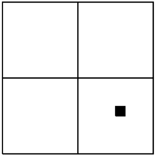

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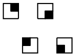

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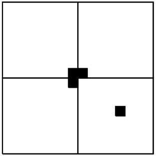

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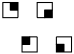

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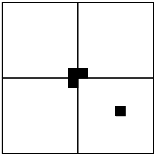

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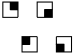
Flipping the orientation can leave hole anywhere. 

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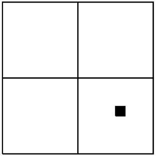

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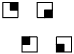

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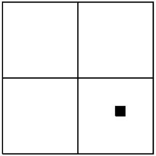

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Strong Induction.

Theorem: Every natural number $n > 1$ can be written as a (possibly trivial) product of primes.

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Strong Induction Principle: If $P(0)$ and

$$(\forall k \in N)((P(0) \wedge \ldots \wedge P(k)) \implies P(k+1)),$$

then $(\forall k \in N)(P(k))$.

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Note: can do with different definition of smallest

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Tournaments have short cycles

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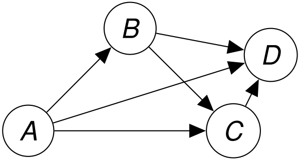

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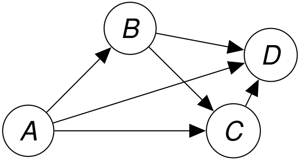

Theorem: Any tournament that has a cycle has a cycle of length 3.

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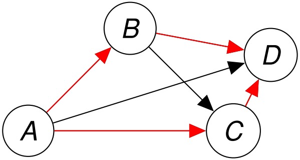

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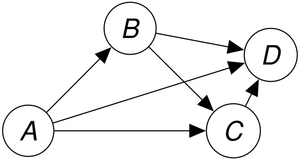

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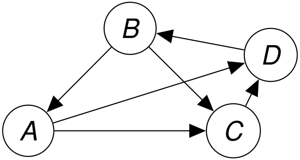

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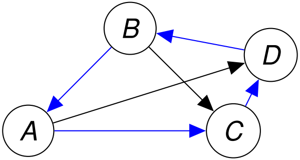

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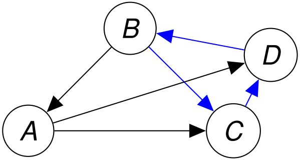

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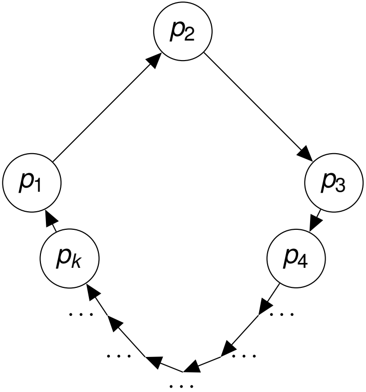

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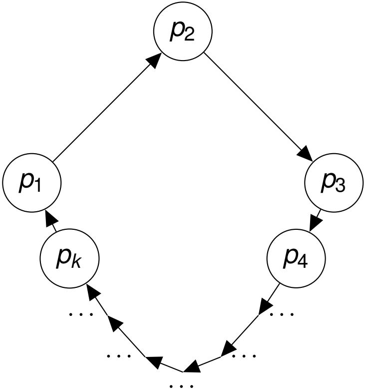

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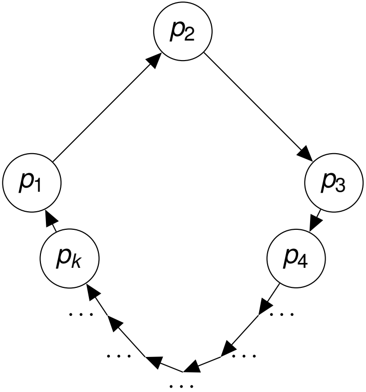

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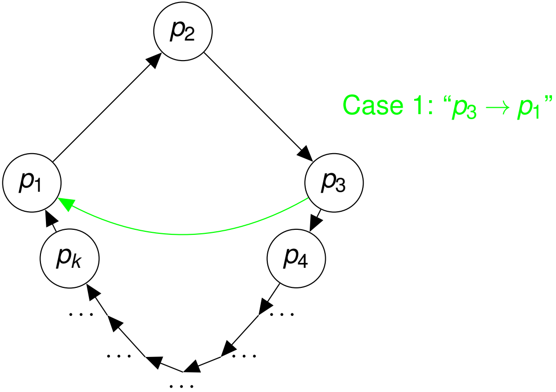

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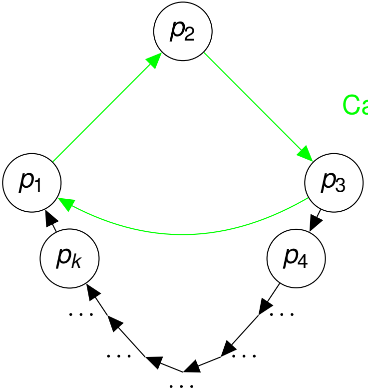

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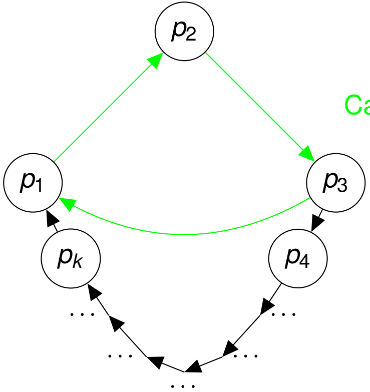

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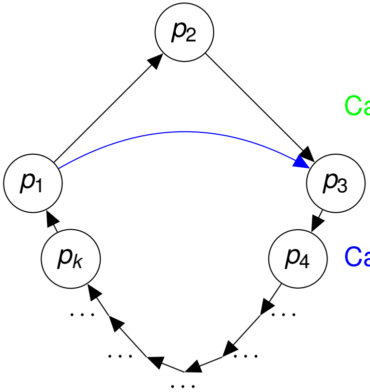

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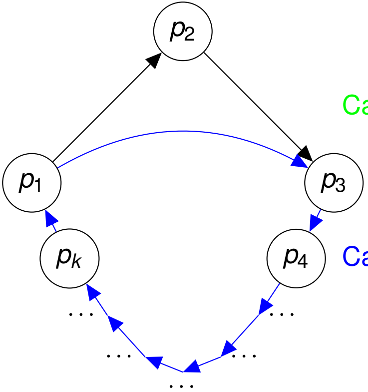

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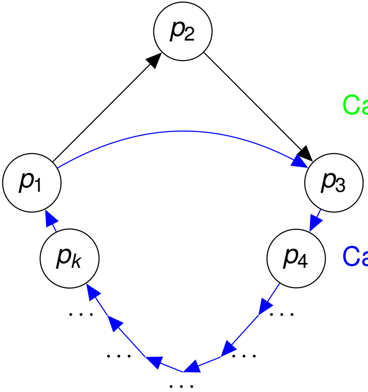

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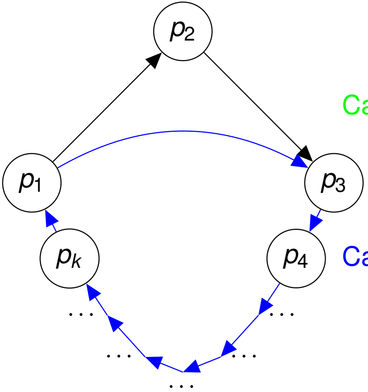

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Def: A **Hamiltonian path**: a sequence $p_1, \ldots, p_n$, $(\forall i, 0 \leq i < n)$ $p_i \to p_{i+1}$.

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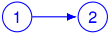

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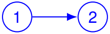

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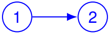

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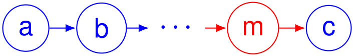

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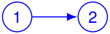

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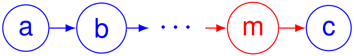

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$\square$

Horses of the same color...

Theorem: All horses have the same color.

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Base Case: $P(1)$ - trivially true.

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As we will see, it is more subtle to catch errors in proofs of correct theorems!!

Sad Islanders...

Island with 100 possibly blue-eyed and green-eyed inhabitants.

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Result: Poll.

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Why?

They know induction.

Thm: If there are n villagers with green eyes they do ritual on day n.

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Induction step:
On day $n+1$, a green eyed person sees n people with green eyes.

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Thm: If there are n villagers with green eyes they do ritual on day n.

Proof:
Base: $n = 1$. Person with green eyes does ritual on day 1.

Induction hypothesis:
If n people with green eyes, they would do ritual on day n.

Induction step:
On day $n + 1$, a green eyed person sees n people with green eyes.

But they didn't do the ritual.

They know induction.

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Wait! Visitor added no information.

Common Knowledge.

Using knowledge about what other people's knowledge (your eye color) is.

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On day 1, everyone knows everyone sees more than zero.

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On day 99, everyone knows no one sees 98

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On day 99, everyone knows no one sees 98
since everyone knows everyone else does not see 97...

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On day 100,

Common Knowledge.

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On day 100, ...uh oh!

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Another example:

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Another example:
Emperor's new clothes!
No one knows other people see that he has no clothes.

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On day 99, everyone knows no one sees 98
  since everyone knows everyone else does not see 97...

On day 100, ...uh oh!

Another example:
Emperor's new clothes!
  No one knows other people see that he has no clothes.
  Until kid points it out.

Summary: principle of induction.

Today: More induction.

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$(P(0)$

Summary: principle of induction.

Today: More induction.

$(P(0) \wedge ((\forall k \in N)(P(k) \implies P(k+1))))$

Summary: principle of induction.

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$$(P(0) \wedge ((\forall k \in N)(P(k) \implies P(k+1)))) \implies (\forall n \in N)(P(n))$$

Summary: principle of induction.

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$(P(0) \wedge ((\forall k \in N)(P(k) \implies P(k+1)))) \implies (\forall n \in N)(P(n))$

Statement to prove: $P(n)$ for n starting from n_0

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Base Case: Prove $P(n_0)$.

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Ind. Step: Prove.

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Strengthen theorem statement.

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Strengthen theorem statement.
Sum of first n odds is n^2.

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Hole anywhere.

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Hole anywhere.
Not same as strong induction.

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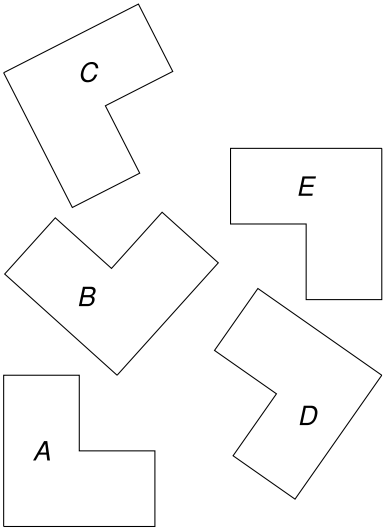
Induction $\equiv$ Recursion.

Tiling Cory Hall Courtyard.



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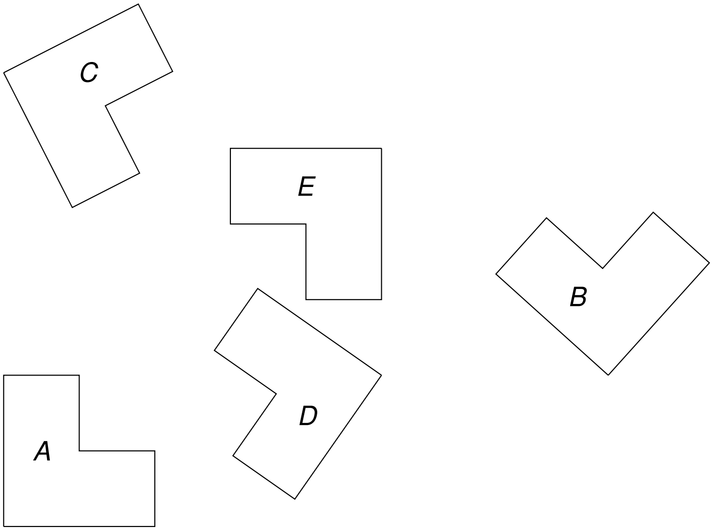

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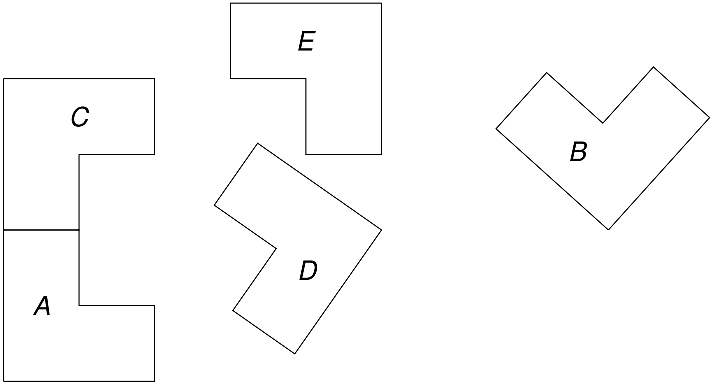

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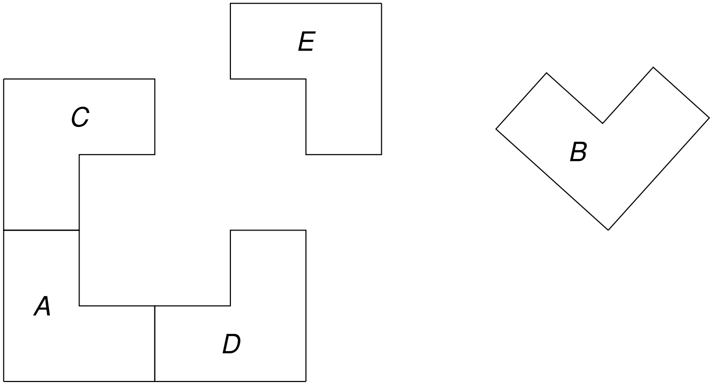

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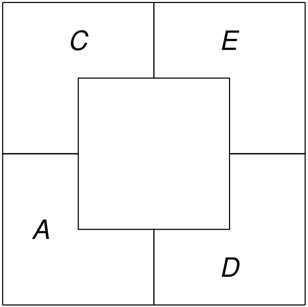

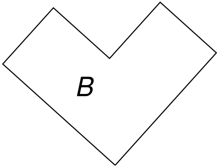

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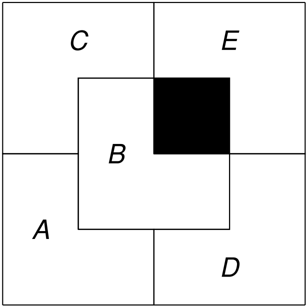

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