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4. Euclid's GCD Algorithm.
   A little tricky here!

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Hypercube: $\Theta(1)$.

Surface Area is roughly at least the volume!

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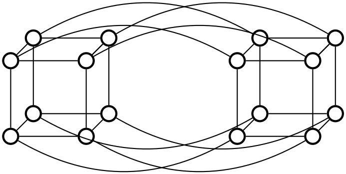

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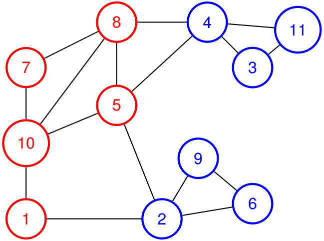
Restatement: for any cut in the hypercube, the number of cut edges is at least the size of the small side.

Cuts in graphs.



S is red, $V - S$ is blue.

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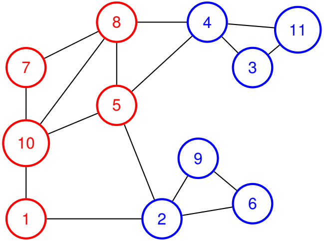

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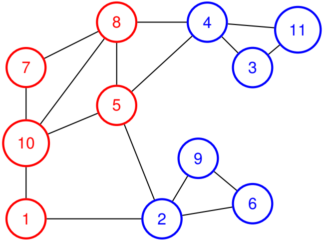

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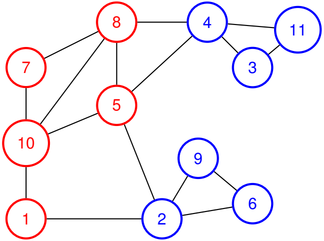

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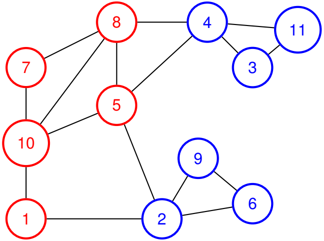

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Number of edges between red and blue. 4.

Hypercube: any cut that cuts off x nodes has $\geq x$ edges.

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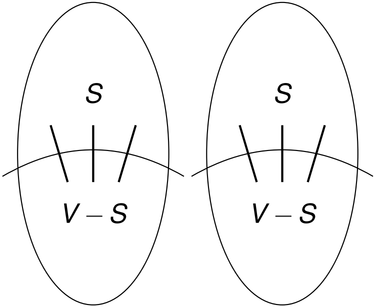

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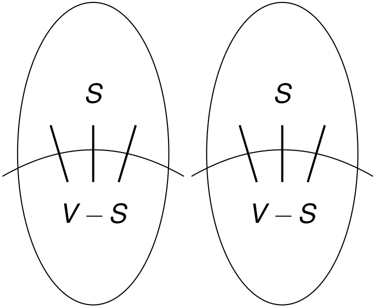

Case 2: Count inside and across.

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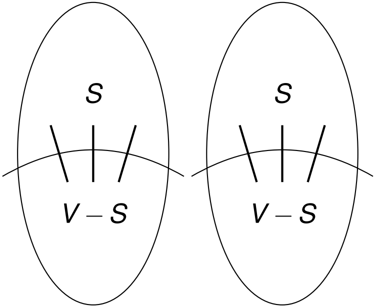

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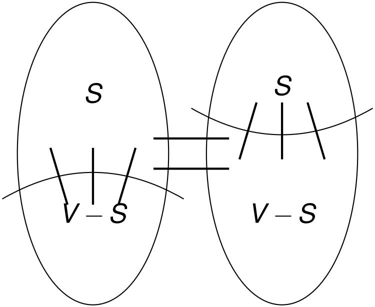

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Total cut edges $\ge |S_0| + |S_1| = |S|$.

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Induction Step. Case 2.

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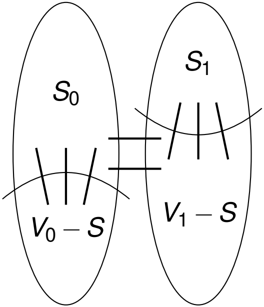
Proof: Induction Step. Case 2.

$$|S_0| \geq |V_0|/2.$$



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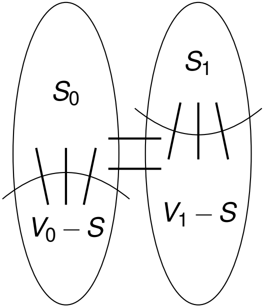

$|S_0| \geq |V_0|/2$.

Recall Case 1: $|S_0|, |S_1| \leq |V|/2$

$|S_1| \leq |V_1|/2$ since $|S| \leq |V|/2$.

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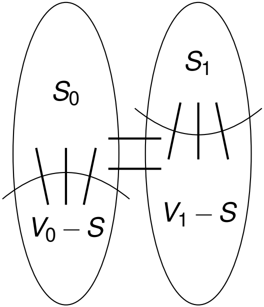

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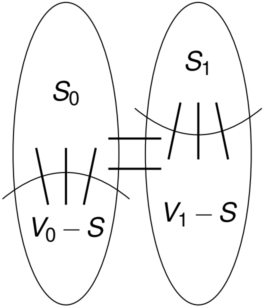

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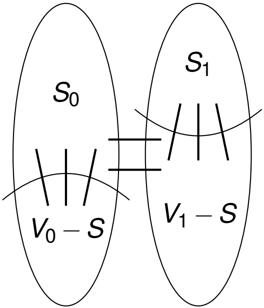

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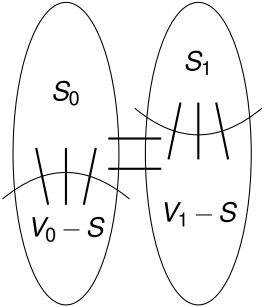

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Edges in E_x connect corresponding nodes.

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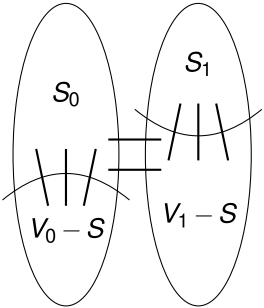

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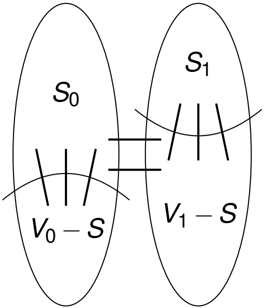

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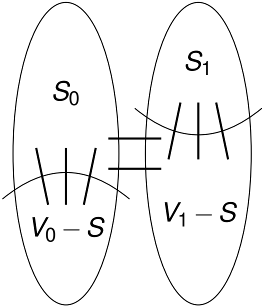

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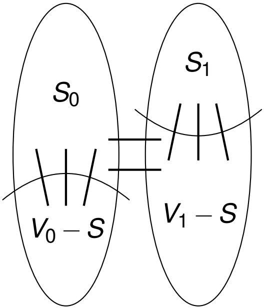

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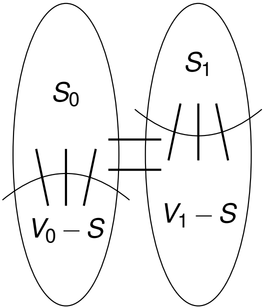

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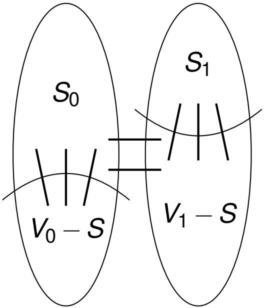

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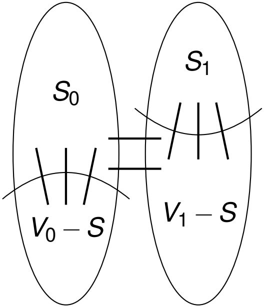

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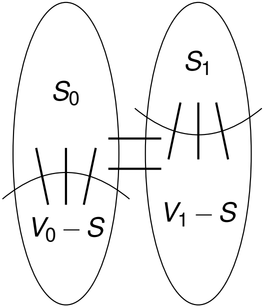

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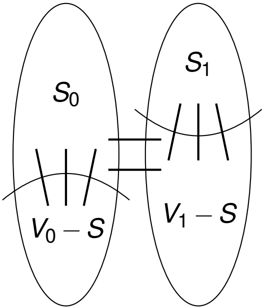

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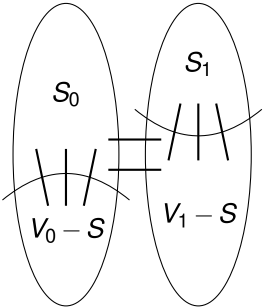

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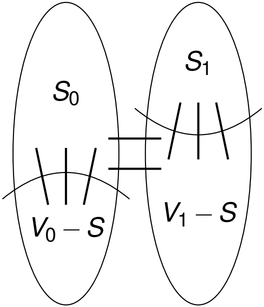

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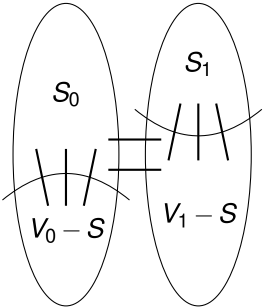

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Also, case 3 where $|S_1| \geq |V|/2$ is symmetric.

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The cuts in the hypercubes are exactly the transitions from 0 sets to 1 set on boolean functions on $\{0,1\}^n$.

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Central object of study.

Modular Arithmetic.

Applications: cryptography, error correction.

Key idea for modular arithmetic.

Theorem: If $d|x$ and $d|y$, then $d|(y-x)$.

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$\square$

Theorem: Every number $n \geq 2$ can be represented as a product of primes.

Proof: Either prime, or $n = a \times b$, and use strong induction. $\square$
(Uniqueness? Later.)

Next Up.

Modular Arithmetic.

Clock Math

If it is 1:00 now.

Clock Math

If it is 1:00 now.
What time is it in 2 hours?

Clock Math

If it is 1:00 now.
What time is it in 2 hours? 3:00!

Clock Math

If it is 1:00 now.

What time is it in 2 hours? 3:00!

What time is it in 5 hours?

Clock Math

If it is 1:00 now.

What time is it in 2 hours? 3:00!

What time is it in 5 hours? 6:00!

Clock Math

If it is 1:00 now.
What time is it in 2 hours? 3:00!
What time is it in 5 hours? 6:00!
What time is it in 15 hours?

Clock Math

If it is 1:00 now.
What time is it in 2 hours? 3:00!
What time is it in 5 hours? 6:00!
What time is it in 15 hours? 16:00!

Clock Math

If it is 1:00 now.

- What time is it in 2 hours? 3:00!
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16 is the "same as 4" with respect to a 12 hour clock system.

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Clock time equivalent up to to addition/subtraction of 12.

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If it is 1:00 now.

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What time is it in 100 hours?

Clock Math

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Actually 4:00.

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What time is it in 100 hours? 101:00!

Clock Math

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What time is it in 100 hours? 101:00! or 5:00.

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 $101 = 12 \times 8 + 5$.

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5 is the same as 101 for a 12 hour clock system.

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Custom is only to use the representative in $\{12, 1, \ldots, 11\}$

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(Almost remainder, except for 12 and 0 are equivalent.)

Day of the week.

Today is Thursday.

Day of the week.

Today is Thursday.

What day is it a year from now?

Day of the week.

Today is Thursday.

What day is it a year from now? on September 17, 2021?

Day of the week.

Today is Thursday.

What day is it a year from now? on September 17, 2021?

Number days.

Day of the week.

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0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

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Today: day 4.

Day of the week.

Today is Thursday.

- What day is it a year from now? on September 17, 2021?
  - Number days.
    - 0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 4.

- 5 days from now.

Day of the week.

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    Number days.
      0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 4.
  5 days from now. day 9

Day of the week.

Today is Thursday.

What day is it a year from now? on September 17, 2021?

Number days.

0 for Sunday, 1 for Monday, ..., 6 for Saturday.

Today: day 4.

5 days from now. day 9 or day 2

Day of the week.

Today is Thursday.
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    Number days.
      0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 4.
  5 days from now. day 9 or day 2 or Tuesday.

Day of the week.

Today is Thursday.

What day is it a year from now? on September 17, 2021?

Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 4.

5 days from now. day 9 or day 2 or Tuesday.

25 days from now.

Day of the week.

Today is Thursday.

What day is it a year from now? on September 17, 2021?

Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 4.

5 days from now. day 9 or day 2 or Tuesday.

25 days from now. day 29

Day of the week.

Today is Thursday.

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Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

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5 days from now. day 9 or day 2 or Tuesday.

25 days from now. day 29 or day 1.

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What day is it a year from now?
Next year is not a leap year.

Day of the week.

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What day is it a year from now?
Next year is not a leap year. So 365 days from now.

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What day is it a year from now?
Next year is not a leap year. So 365 days from now.
Day 4+365 or day 369.

Day of the week.

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Smallest representation:

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What day is it a year from now?

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Day 4+365 or day 369.

Smallest representation:

subtract 7 until smaller than 7.

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What day is it a year from now?
  Next year is not a leap year. So 365 days from now.
  Day 4+365 or day 369.
Smallest representation:
  subtract 7 until smaller than 7.
  divide and get remainder.

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  Next year is not a leap year. So 365 days from now.
  Day 4+365 or day 369.
Smallest representation:
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  divide and get remainder.
  369/7

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subtract 7 until smaller than 7.

divide and get remainder.

369/7 leaves quotient of 52 and remainder 3.

Day of the week.

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 Next year is not a leap year. So 365 days from now.
 Day 4+365 or day 369.
Smallest representation:
  subtract 7 until smaller than 7.
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 369/7 leaves quotient of 52 and remainder 3. $369 = 7(52) + 5$

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or September 18, 2020 is a Friday.

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Years and years...

80 years from now?

Years and years...

80 years from now? 20 leap years.

Years and years...

80 years from now? 20 leap years. 366×20 days

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years.

Years and years...

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Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$.

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7?

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7?

Years and years...

80 years from now? 20 leap years. 366×20 days

60 regular years. 365×60 days

Today is day 4.

It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.

What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.

What is remainder of 365 when dividing by 7? 1

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1

Years and years...

80 years from now? 20 leap years. 366×20 days
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Today is day 4.
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Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 4.

Years and years...

80 years from now? 20 leap years. 366×20 days
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Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 4.
Get Day: $4 + 2 \times 20 + 1 \times 60$

Years and years...

80 years from now? 20 leap years. 366×20 days

60 regular years. 365×60 days

Today is day 4.

It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.

What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.

What is remainder of 365 when dividing by 7? 1

Today is day 4.

Get Day: $4 + 2 \times 20 + 1 \times 60 = 104$

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 4.
Get Day: $4 + 2 \times 20 + 1 \times 60 = 104$
Remainder when dividing by 7?

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 4.
Get Day: $4 + 2 \times 20 + 1 \times 60 = 104$
Remainder when dividing by 7? $104 = 14 \times 7$

Years and years...

80 years from now? 20 leap years. 366×20 days
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Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 4.
Get Day: $4 + 2 \times 20 + 1 \times 60 = 104$
Remainder when dividing by 7? $104 = 14 \times 7 + 6$.

Years and years...

80 years from now? 20 leap years. 366×20 days
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Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 4.
Get Day: $4 + 2 \times 20 + 1 \times 60 = 104$
Remainder when dividing by 7? $104 = 14 \times 7 + 6$.
Or September 18, 2100 is Saturday!

Years and years...

80 years from now? 20 leap years. 366×20 days
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Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

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What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 4.
Get Day: $4 + 2 \times 20 + 1 \times 60 = 104$
Remainder when dividing by 7? $104 = 14 \times 7 + 6$.
Or September 18, 2100 is Saturday!

Further Simplify Calculation:

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
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Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 4.
Get Day: $4 + 2 \times 20 + 1 \times 60 = 104$
Remainder when dividing by 7? $104 = 14 \times 7 + 6$.
Or September 18, 2100 is Saturday!

Further Simplify Calculation:
20 has remainder 6 when divided by 7.

Years and years...

80 years from now? 20 leap years. 366×20 days

60 regular years. 365×60 days

Today is day 4.

It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.

What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.

What is remainder of 365 when dividing by 7? 1

Today is day 4.

Get Day: $4 + 2 \times 20 + 1 \times 60 = 104$

Remainder when dividing by 7? $104 = 14 \times 7 + 6$.

Or September 18, 2100 is Saturday!

Further Simplify Calculation:

20 has remainder 6 when divided by 7.

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Today is day 4.
Get Day: $4 + 2 \times 20 + 1 \times 60 = 104$
Remainder when dividing by 7? $104 = 14 \times 7 + 6$.
Or September 18, 2100 is Saturday!

Further Simplify Calculation:
20 has remainder 6 when divided by 7.
60 has remainder 4 when divided by 7.
Get Day: $2 + 2 \times 6 + 1 \times 4 = 18$.

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 4.
Get Day: $4 + 2 \times 20 + 1 \times 60 = 104$
Remainder when dividing by 7? $104 = 14 \times 7 + 6$.
Or September 18, 2100 is Saturday!

Further Simplify Calculation:
20 has remainder 6 when divided by 7.
60 has remainder 4 when divided by 7.
Get Day: $2 + 2 \times 6 + 1 \times 4 = 18$.
Or Day 6.

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 4.
It is day $4 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 4.
Get Day: $4 + 2 \times 20 + 1 \times 60 = 104$
Remainder when dividing by 7? $104 = 14 \times 7 + 6$.
Or September 18, 2100 is Saturday!

Further Simplify Calculation:
20 has remainder 6 when divided by 7.
60 has remainder 4 when divided by 7.
Get Day: $2 + 2 \times 6 + 1 \times 4 = 18$.
Or Day 6. September 18, 2100 is Saturday.

Years and years...

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60 regular years. 365×60 days

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Hmm.

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Today is day 4.

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Remainder when dividing by 7? $104 = 14 \times 7 + 6$.

Or September 18, 2100 is Saturday!

Further Simplify Calculation:

20 has remainder 6 when divided by 7.

60 has remainder 4 when divided by 7.

Get Day: $2 + 2 \times 6 + 1 \times 4 = 18$.

Or Day 6. September 18, 2100 is Saturday.

"Reduce" at any time in calculation!

Modular Arithmetic: refresher.

x **is congruent to** y **modulo** m or "$x \equiv y \pmod{m}$"
if and only if $(x - y)$ is divisible by m.

Modular Arithmetic: refresher.

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...or x and y have the same remainder w.r.t. m.

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...or $x = y + km$ for some integer k.

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Mod 7 equivalence classes:

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Mod 7 equivalence classes:
$\{\ldots, -7, 0, 7, 14, \ldots\}$

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Mod 7 equivalence classes:
$\{\ldots,-7,0,7,14,\ldots\}$ $\{\ldots,-6,1,8,15,\ldots\}$

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Useful Fact: Addition, subtraction, multiplication can be done with any equivalent x and y.

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or " $a \equiv c \pmod{m}$ and $b \equiv d \pmod{m}$

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or " $a \equiv c \pmod{m}$ and $b \equiv d \pmod{m}$
$\implies a + b \equiv c + d \pmod{m}$ and $a \cdot b = c \cdot d \pmod{m}$"

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or " $a \equiv c \pmod{m}$ and $b \equiv d \pmod{m}$
$\implies a+b \equiv c+d \pmod{m}$ and $a \cdot b = c \cdot d \pmod{m}$"

Proof: If $a \equiv c \pmod{m}$, then $a = c + km$ for some integer k.

Modular Arithmetic: refresher.

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or " $a \equiv c \pmod{m}$ and $b \equiv d \pmod{m}$
$\implies a+b \equiv c+d \pmod{m}$ and $a \cdot b = c \cdot d \pmod{m}$"

Proof: If $a \equiv c \pmod{m}$, then $a = c + km$ for some integer k.
If $b \equiv d \pmod{m}$, then $b = d + jm$ for some integer j.

Modular Arithmetic: refresher.

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or " $a \equiv c \pmod{m}$ and $b \equiv d \pmod{m}$
$\implies a+b \equiv c+d \pmod{m}$ and $a \cdot b = c \cdot d \pmod{m}$"

Proof: If $a \equiv c \pmod{m}$, then $a = c + km$ for some integer k.
If $b \equiv d \pmod{m}$, then $b = d + jm$ for some integer j.
Therefore,

Modular Arithmetic: refresher.

x **is congruent to** y **modulo** m or "$x \equiv y \pmod{m}$"
if and only if $(x - y)$ is divisible by m.
...or x and y have the same remainder w.r.t. m.
...or $x = y + km$ for some integer k.

Mod 7 equivalence classes:
$\{\ldots,-7,0,7,14,\ldots\}$ $\{\ldots,-6,1,8,15,\ldots\}$ ...

Useful Fact: Addition, subtraction, multiplication can be done with any equivalent x and y.

or " $a \equiv c \pmod{m}$ and $b \equiv d \pmod{m}$
$\implies a+b \equiv c+d \pmod{m}$ and $a \cdot b = c \cdot d \pmod{m}$"

Proof: If $a \equiv c \pmod{m}$, then $a = c + km$ for some integer k.
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x **is congruent to** y **modulo** m or "$x \equiv y \pmod{m}$"
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Can calculate with representative in $\{0,\ldots,m-1\}$.

Notation

$x \pmod{m}$ or $\bmod(x, m)$

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Work in this system.

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Work in this system.
$a \equiv b \pmod{m}$.

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Says two integers a and b are equivalent modulo m.

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$6 \equiv$

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$6 \equiv 3 + 3$

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Modulus is m

$6 \equiv 3 + 3 \equiv 3 + 10$

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Modulus is m

$6 \equiv 3 + 3 \equiv 3 + 10 \pmod{7}$.

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Inverses and Factors.

Division: multiply by multiplicative inverse.

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Check!

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Check! $4(3) = 12 = 5 \pmod{7}$.

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"Common factor of 4"

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"Common factor of 4" $\implies$
$8k - 12\ell$ is a multiple of four for any ℓ and k $\implies$

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"Common factor of 4" $\implies$
$8k - 12\ell$ is a multiple of four for any ℓ and k $\implies$
$8k \not\equiv 1 \pmod{12}$ for any k.

Greatest Common Divisor and Inverses.

Thm:
If greatest common divisor of x and m, $\gcd(x, m)$, is 1, then x has a multiplicative inverse modulo m.

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Claim: The set $S = \{0x, 1x, \ldots, (m-1)x\}$ contains $y \equiv 1 \mod m$ if all distinct modulo m.

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Each of m numbers in S correspond to different one of m equivalence classes modulo m.

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Not distinct. Common factor 2.

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Proof Sketch: The set $S = \{0x, 1x, \dots, (m-1)x\}$ contains
$y \equiv 1 \mod m$ if all distinct modulo m.

... $\square$

For $x = 4$ and $m = 6$. All products of 4...
$S = \{0(4), 1(4), 2(4), 3(4), 4(4), 5(4)\} = \{0, 4, 8, 12, 16, 20\}$
reducing $\pmod{6}$
$S = \{0, 4, 2, 0, 4, 2\}$
Not distinct. Common factor 2. Can't be 1. No inverse.

For $x = 5$ and $m = 6$.
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Very different for elements with inverses.

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Not a bijection.

Finding inverses.

How to find the inverse?

Finding inverses.

How to find the inverse?

How to find **if** x has an inverse modulo m?

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Find gcd (x, m).

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Greater than 1?

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Greater than 1? No multiplicative inverse.

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Inverses

Next up.

Inverses

Next up.

Inverses

Next up.

Euclid's Algorithm.

Inverses

Next up.

Euclid's Algorithm.

Runtime.

Inverses

Next up.

Euclid's Algorithm.
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Euclid's Extended Algorithm.

Refresh

Does 2 have an inverse mod 8?

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Any multiple of 2 is 2 away from $0+8k$ for any $k \in \mathbb{N}$.

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$3 = gcd(6,9)$!

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Does 6 have an inverse mod 9? No.
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x has an inverse modulo m if and only if
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Refresh

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Division? Multiply by multiplicative inverse.

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Division? Multiply by multiplicative inverse.
$a \pmod{N}$ has multiplicative inverse, $a^{-1} \pmod{N}$.

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